College Algebra — Joysheet 11
MAT 140, Spring 2023 — D. Ivanšić

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Covers: 5.4-5.6 *Show all your work!*

1. (5pts) If $\log_a 7 = 0.6868$ and $\log_a 3 = 0.3878$, calculate:

$\log_a 21 = \log_a(7\cdot 3)$

$= \log_a 7 + \log_a 3$

$= 0.6868 + 0.3878$

$= 1.0746$

$\log_a \frac{9}{7} = \log_a 9 - \log_a 7$

$= \log_a 3^2 - \log_a 7 = 2\log_a 3 - \log_a 7$

$= 2\cdot 0.3878 - 0.6868$

$= 0.0888$

2. (11pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$\log_4(64x^3y^7) = \log_4 64 + \log_4 x^3 + \log_4 y^7$

$= 3 + 3\log_4 x + 7\log_4 y$

$4^? = 64$

$\log \frac{1000\sqrt[3]{x^5}y^6}{x^{\frac{2}{3}}z^3} = \log 1000 + \log x^{\frac{5}{3}} + \log y^6 - \log x^{\frac{2}{3}} - \log z^3$

$= 3 + \frac{5}{3}\log x + 6\log y - \frac{2}{3}\log x - 3\log z$

$= 3 + \log x + 6\log y - 3\log z$

3. (12pts) Write as a single logarithm. Simplify if possible.

$3\log_5(2x^4) + 4\log_5 x^2 - 2\log_5(2y^3) = \log_5(2x^4)^3 + \log_5(x^2)^4 - \log_5(2y^3)^2$

$= \log_5 \frac{(2x^4)^3(x^2)^4}{(2y^3)^2} = \log_5 \frac{8x^{12}x^8}{4y^6} = \log_5 \frac{2x^{20}}{y^6}$

$\log_3(x^2-16) + 2\log_3(x+4) - \log_3(x^2-7x+12) =$

$= \log_3((x-4)(x+4)) - \log_3(x+4)^2 - \log_3((x-4)(x-3))$

$= \log_3 \frac{\cancel{(x-4)}(x+4)(x+4)^2}{\cancel{(x-4)}(x-3)} = \log_3 \frac{(x+4)^3}{x-3}$

4. (3pts) Simplify. $\log_4 4^{2x-1} = 2x-1$ $\quad e^{\ln(3x)} = 3x$

Solve the equations.

5. (5pts) $7^{2x-3} = 49^{3x}$

$7^{2x-3} = (7^2)^{3x}$

$7^{2x-3} = 7^{6x}$

$2x-3 = 6x$

$-3 = 4x$

$x = -\frac{3}{4}$

6. (7pts) $3^{x-1} = 4^{2x}$

$\ln 3^{x-1} = \ln 4^{2x}$

$(x-1)\ln 3 = 2x \ln 4$

$x\ln 3 - \ln 3 = 2x \ln 4$

$x \ln 3 - 2x\ln 4 = \ln 3$

$x(\ln 3 - 2\ln 4) = \ln 3$

$x = \dfrac{\ln 3}{\ln 3 - 2\ln 4} \approx -0.656289$

7. (5pts) A tire shop bought a car jack for \$1,400. The value of the car jack each year is 85% of the value of the year before, so after t years its value is given by the function $V(t) = 1400 \cdot 0.85^t$. When will the value of the car jack be \$400?

$1400 \cdot 0.85^t = 400$

$0.85^t = \frac{400}{1400} = \frac{2}{7}$

$\ln 0.85^t = \ln \frac{2}{7}$

$t \ln 0.85 = \ln\frac{2}{7}$

$t = \dfrac{\ln\frac{2}{7}}{\ln 0.85} = 7.708413$

In about 8 years.

8. (12pts) The town of Renewal had 34,000 inhabitants in 2015 and 41,000 in 2020. Assume the population of Renewal grows exponentially.
a) Write the function describing the number $P(t)$ of people in Renewal t years after 2015. Then find the exponential growth rate for this population.
b) Graph the function.
c) According to this model, when will the population reach 50,000?

a) $P(t) = 34e^{kt}$

$41 = P(5) = 34e^{k\cdot 5}$

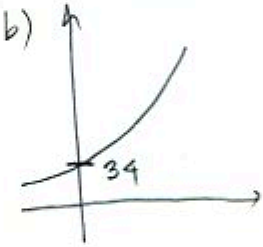
$\frac{41}{34} = e^{k\cdot 5} \quad | \ln$

$\ln\frac{41}{34} = \ln e^{k\cdot 5}$

$\ln\frac{41}{34} = 5k$

$k = \dfrac{\ln\frac{41}{34}}{5} = 0.0374423$

b)



c) $34e^{0.0374423t} = 50$

$e^{0.037\ldots t} = \frac{50}{34} = \frac{25}{17}$

$\ln e^{0.037\ldots t} = \ln\frac{25}{17}$

$0.037\ldots t = \ln\frac{25}{17}$

$t = \dfrac{\ln\frac{25}{17}}{0.0374423} = 10.300179$

About 10 years from 2015, in 2025.