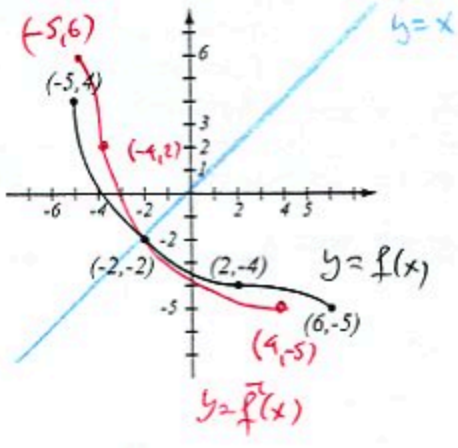
College Algebra — Joysheet 10
MAT 140, Spring 2023 — D. Ivanšić

Name: Saul Ocean

Covers: 5.1–5.3 *Show all your work!*

1. (6pts) The graph of a function f is given.
a) Is this function one-to-one? Justify.
b) If the function is one-to-one, find the graph of f^{-1}, labeling the relevant points.



a) Yes, it passes the horizontal line test,

2. (12pts) Let $f(x) = \dfrac{2x}{x+3}$. Find the formula for f^{-1}. Find the ranges of f and f^{-1}.

$y = \dfrac{2x}{x+3}$

$y(x+3) = 2x$

$yx + 3y = 2x$

$yx - 2x = -3y$

$(y-2)x = -3y$

$x = \dfrac{-3y}{y-2} = \dfrac{3y}{2-y}$

$f^{-1}(y) = \dfrac{3y}{2-y}$

Range f = domain f^{-1}

Can't have $2-y=0$

$y=2$

$(-\infty, 2) \cup (2, \infty)$

Range f^{-1} = domain f

Can't have $x+3=0$

$x=-3$

$(-\infty, -3) \cup (-3, \infty)$

3. (8pts) Evaluate without using the calculator. For each problem, write the question you should ask yourself in order to find the logarithms.

$\log_5 125 = 3$ $\quad \log \frac{1}{100} = -2$ $\quad \log_{27} 3 = \frac{1}{3}$ $\quad \log_{a^3} a^{15} = 5$

$5^? = 125$ $\quad 10^? = \frac{1}{100} = \frac{1}{10^2} = 10^{-2}$ $\quad 27^? = 3 = \sqrt[3]{27} = 27^{\frac{1}{3}}$ $\quad (a^3)^? = a^{15}$

$3 \cdot ? = 15$

4. (4pts) Use the change-of-base formula and your calculator to find $\log_7 0.7$ with accuracy 6 decimal places. Show how you obtained your number.

$\log_7 0.7 = \dfrac{\log 0.7}{\log 7} \approx -0.183295$

5. (12pts) Investigate the effect of increased frequency of compounding: for a deposit of \$1,000 and annual interest rate of 4.14%, calculate the amount in the account after 1 year for the frequencies of compounding below.
a) Write the general formula for the amount, replacing the variables by numbers, if known.
b) Use the table feature on your calculator to quickly compute amounts after 1 year.
c) Does compounding more often make a big difference?

Frequency: every	n	Amount after 1 year
year	1	1041.40
quarter	4	1042.05
month	12	1042.19
day	365	1042.27
hour	$365 \cdot 24 = 8760$	1042.27
second	$8760 \cdot 3600$	1042.27

a) $A = P\left(1+\frac{r}{n}\right)^{nt}$

$= 1000\left(1+\frac{0.0414}{n}\right)^{n \cdot 1}$

c) There is no difference once the compounding frequency goes past daily

6. (3pts) Find the domain of $f(x) = \ln(7-2x)$.

Must have $7-2x > 0$ $\quad x < \frac{7}{2}$ $\quad (-\infty, \frac{7}{2})$

$7 > 2x$

7. (8pts) The expression that converts temperatures in degrees Celsius (C) to degrees Fahrenheit (F) is $F(C) = 32 + \frac{9}{5}C$.
a) Find temperatures in degrees Fahrenheit for $-5°$C, and $20°$C .
b) Find a formula for the inverse function and explain what it represents.
c) What temperatures in degrees Celsius correspond to 41°F and 95°F?

a) $F(-5) = 32 + \frac{9}{5}\cdot(-5) = 32-9 = 23°$

$F(20) = 32 + \frac{9}{5}\cdot 20 = 32+36 = 68°$

b) $F = 32 + \frac{9}{5}C$

$F-32 = \frac{9}{5}C \quad | \cdot \frac{5}{9}$

$C = \frac{5}{9}(F-32)$

gives the expression that converts temperature in Fahrenheit (F) into temperature in Celsius

c) $C = \frac{5}{9}(41-32) = \frac{5}{9}\cdot 9 = 5$

$C = \frac{5}{9}(95-32) =$

$= \frac{5}{9}\cdot 63 = 35$

8. (7pts) Using transformations, draw the graph of $f(x) = 4 - e^x$. Explain how you transform the graph of a basic function in order to get the graph of f. Show at least one point on the graph, and asymptotes to the graph, if any.