College Algebra — Joysheet 8
MAT 140, Spring 2023 — D. Ivanšić

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Covers: 3.1, 3.2, 3.3 Show all your work!

Simplify, so that the answer is in form $a + bi$.

1. (4pts) $2i(-3+5i) - (5i)^2 = -6i + 10i^2 - 25i^2$

$= -6i - 10 + 25 = 15 - 6i$

2. (6pts) $\frac{2+3i}{2-i} = \frac{2+3i}{2-i} \cdot \frac{2+i}{2+i} = \frac{4+6i+2i+3i^2}{2^2-i^2} = \frac{4+8i-3}{4+1} = \frac{1+8i}{5}$

$= \frac{1}{5} + \frac{8}{5}i$

3. (4pts) Simplify and justify your answer.

$i^{58} = i^{56} \cdot i^2 = (i^4)^{19} \cdot i^2 = 1 \cdot (-1) = -1$

$56 = 4 \cdot 19$

4. (8pts) The number of plants for sale in a nursery is described by the function $N(x) = x^2 - 16x + 100$, where x is the number of days after April 22nd.

a) On what dates did the nursery have 52 plants for sale?

b) On what date did the number of plants for sale bottom out?

a) $x^2 - 16x + 100 = 52$

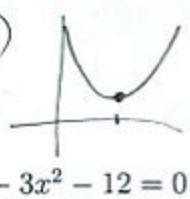
$x^2 - 16x + 48 = 0$

$(x-12)(x-4) = 0$

$x = 12, 4$

12 days afte Apr 22nd is May 4th

4 days aftk Apr 22nd is Apr 26th

b) 

$h = -\frac{b}{2a} = -\frac{-16}{2 \cdot 1} = 8$

8 days afte Apr. 22nd is Apr. 30th.

5. (8pts) Solve the equation: $x^4 - 3x^2 - 12 = 0$

$u = x^2$ $u^2 - 3u - 12 = 0$ $9+48$

$u = \frac{-(-3) \pm \sqrt{(-3)^2 - 4 \cdot 1 \cdot (-12)}}{2 \cdot 1}$

$= \frac{3 \pm \sqrt{57}}{2}$

$x^2 = \frac{3 \pm \sqrt{57}}{2}$

$x = \pm\sqrt{\frac{3+\sqrt{57}}{2}}$ $x = \pm\sqrt{\frac{3-\sqrt{57}}{2}}$

6. (6pts) Solve by completing the square.

$x^2 - 12x + 53 = 0$ $|+6^2$

$x^2 - 2 \cdot x \cdot 6 + 6^2 + 53 = 6^2$ $|-53$

$(x-6)^2 = 36 - 53$

$(x-6)^2 = -17$

$x - 6 = \pm\sqrt{-17}$

$x = 6 \pm \sqrt{17}\, i$

7. (12pts) The quadratic function $f(x) = x^2 + 4x + 8$ is given. Do the following without using the calculator.

a) Find the x-intercepts of its graph, if any. Find the y-intercept.

b) Find the vertex of the graph.

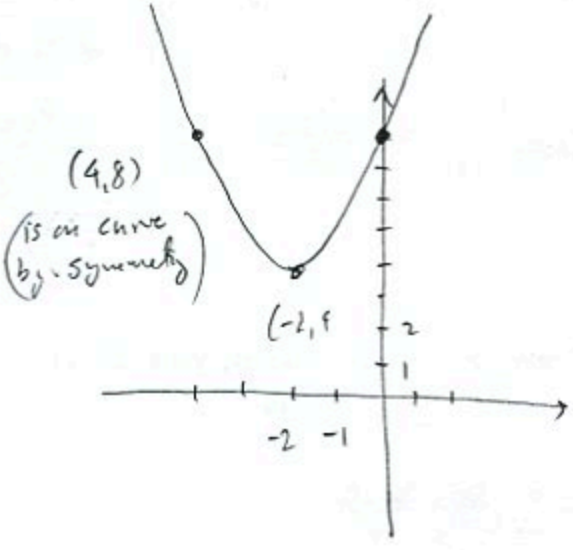
c) Sketch the graph of the function.

a) $x^2+4x+8=0$

$x = \dfrac{-4 \pm \sqrt{4^2-4\cdot 1\cdot 8}}{2\cdot 1} = \dfrac{-4\pm\sqrt{-16}}{2}$

No real solutions, so no x-int

y-int: $f(0) = 8$



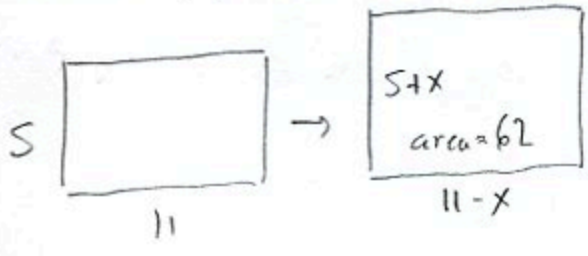
b) $h = -\dfrac{b}{2a} = -\dfrac{4}{2\cdot 1} = -2$

$k = f(-2) = (-2)^2 + 4(-2) + 8$

$= 4 - 8 + 8 = 4$

Vertex: $(-2, 4)$

8. (12pts) We wish to alter a 5 ft × 11 ft plot in the garden by lengthening the 5 ft side and shortening the 11 ft side by the same amount. If the area of the new rectangular plot is to be 62 ft^2, by how much do we change the lengths of the sides?



$(5+x)(11-x) = 62$

$55 - 5x + 11x - x^2 = 62$

$-x^2 + 6x + 55 - 62 = 0$

$-x^2 + 6x - 7 = 0 \quad \cdot(-1)$

$x^2 - 6x + 7 = 0$

$x = \dfrac{-(-6) \pm \sqrt{(-6)^2 - 4\cdot 1\cdot 7}}{2\cdot 1} = \dfrac{6 \pm \sqrt{8}}{2}$ $\sqrt{4\cdot 2}$

$= \dfrac{6 \pm 2\sqrt{2}}{2} = \dfrac{2(3\pm\sqrt{2})}{2} = 3 \pm \sqrt{2}$

Since $3+\sqrt{2}$ and $3-\sqrt{2}$ are positive, they are both solutions,

$3+\sqrt{2} = 4.414214$

$3-\sqrt{2} = 1.585786$