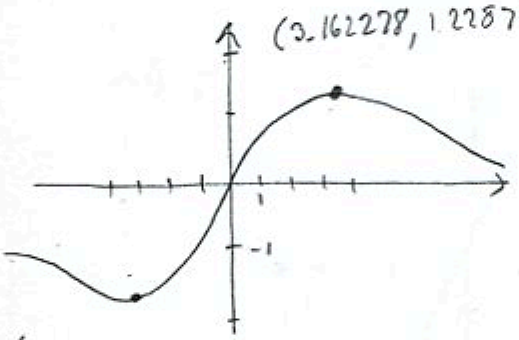
College Algebra — Joysheet 5 **MAT 140, Spring 2023 — D. Ivanšić**	**Name:** Saul Ocean *Covers: 2.1, 2.2, 2.3 Show all your work!*

1. (10pts) Use your calculator to accurately sketch the graph of the function $f(x) = \dfrac{9x}{x^2+x+10}$. (When entering function into calculator, don't forget to put parentheses around the denominator.) Draw the graph here, indicate units on the axes, and solve the problems below with accuracy 6 decimal points.

a) Find the local maxima and minima for this function.

b) State the intervals where the function is increasing and where it is decreasing.



a) Local minimum is $-1.690282 = f(-3.162279)$

Local maximum is $1.228744 = f(3.162278)$

Increasing on $(-3.162279, 3.162278)$

Decreasing on $(-\infty, -3.162279) \cup (3.162278, \infty)$

2. (20pts) Let $f(x) = \dfrac{7}{x-3}$, $g(x) = 4+\sqrt{x}$. Find the following (simplify where possible):

$(f-g)(1) = f(1) - g(1)$

$= \dfrac{7}{1-3} - (4+\sqrt{1}) = -\dfrac{7}{2} - 5 = -\dfrac{7}{2} - \dfrac{10}{2}$

$= -\dfrac{17}{2}$

$(fg)(-4) = f(-4)\cdot g(-4)$ not defined b/c $\sqrt{-4}$ is not defined

$= \dfrac{7}{-4-3}\cdot(4+\sqrt{-4})$

$\dfrac{f}{g}(x) = \dfrac{f(x)}{g(x)}$

$= \dfrac{\frac{7}{x-3}}{4+\sqrt{x}} = \dfrac{7}{x-3}\cdot\dfrac{1}{4+\sqrt{x}} = \dfrac{7}{(x-3)(4+\sqrt{x})}$

$(g\circ f)(4) = g(f(4))$

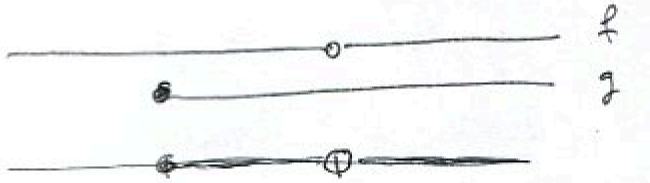
$= g\left(\dfrac{7}{4-3}\right) = g(7) = 4+\sqrt{7}$

$(f\circ g)(x) = f(g(x)) = f(4+\sqrt{x}) = \dfrac{7}{4+\sqrt{x}-3} = \dfrac{7}{1+\sqrt{x}}$

The domain of $(fg)(x)$ in interval notation

Domain f: can't have $x-3=0$
$x=3$

Domain g: must have $x \geq 0$



domain of fg is overlap of domains of f, g

$[0,3) \cup (3,\infty)$

3. (8pts) Consider the function $h(x) = \sqrt{5 - x^2}$ and find **two** different solutions to the following problem: find functions f and g so that $h(x) = f(g(x))$, where neither f nor g are the identity function.

$g(x) = 5 - x^2$ $\quad$ $g(x) = x^2$

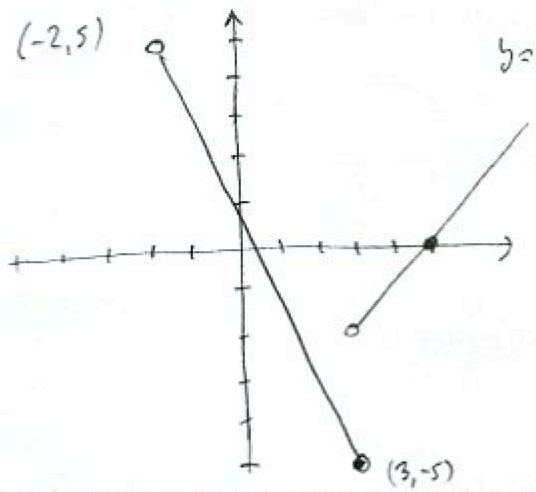
$f(x) = \sqrt{x}$ $\quad$ $f(x) = \sqrt{5-x}$

4. (8pts) Sketch the graph of the piecewise-defined function:

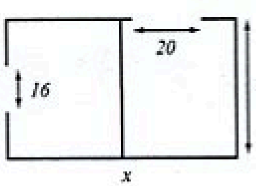
$$f(x) = \begin{cases} -2x+1, & \text{if } -2 < x \leq 3 \\ x-5 & \text{if } x > 3. \end{cases}$$

x	$-2x+1$
-2	5
3	-5

x	$x-5$
3	-2
5	0



5. (14pts) Using a fence, farmer Hilda wishes to enclose a rectangular field with area 20000 square feet, divide into parts and leave 16 ft and 20 ft openings in the fence, as in the picture. She wishes to minimize the total length of the fence.
a) Express the total length of the fence as a function of the length of one of the sides x. What is the domain of this function?
b) Graph the function in order to find the minimum. What are the dimensions of the field for which the total length of the fence is minimal? What is the minimal fence length?



$\ell = x + x - 20 + 2y + y - 16 = 2x + 3y - 36$

$x \cdot y = 20000$, so $y = \dfrac{20000}{x}$

$\ell(x) = 2x + 3 \cdot \dfrac{20000}{x} - 36 = 2x + \dfrac{60000}{x} - 36$

Domain:

Must have:

$x \geq 20$

$y \geq 16$

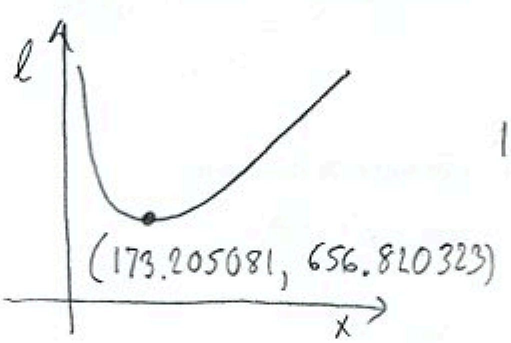
$\dfrac{20000}{x} \geq 16$

$20000 \geq 16x$

$\dfrac{20000}{16} \geq x$ $\quad$ $1250 \geq x$

Domain:

$[20, 1250]$



Dimensions:

x by y.

173.205081 by

Minimal Fence length:

656.820323 ft