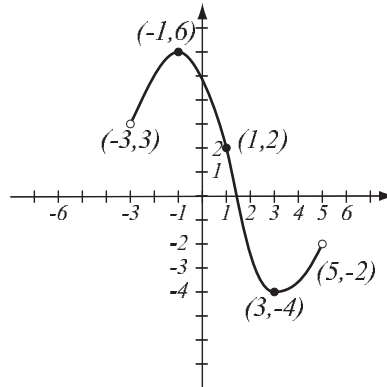


1. (8pts) Use the graph of the function f at right to answer the following questions.

- Find: $f(1) =$ $f(-5) =$
- What is the domain of f ?
- What is the range of f ?
- What are the solutions of the equation $f(x) = 4$?



2. (10pts) Use your calculator to accurately sketch the graph of $y = -x^3 + 8x - 11$.

- Draw the graph on paper and indicate units on the axes.
- Find all the x - and y -intercepts (accuracy: 6 decimal points).

3. (5pts) Draw the line that passes through points $(-2, -3)$ and $(4, 3)$. Then write the equation of the line in form $y = mx + b$.

4. (10pts) Find the equation of the line (in form $y = mx + b$) that is perpendicular to the line $2x - 3y = 9$ and passes through the point $(2, 1)$. Draw both lines.

5. (8pts) Draw the triangle with vertices $A = (-3, -1)$, $B = (7, 4)$ and $C = (1, 7)$. Use distance between two points to determine if this is a right triangle.

6. (9pts) Let $f(x) = \frac{x^2 - 3x}{x + 4}$. Find the following (simplify where appropriate).

$$f(2) =$$

$$f(-4) =$$

$$f(2b) =$$

$$f(x + 1) =$$

7. (7pts) Find the domain of the function below and write it using interval notation.

$$f(x) = \frac{\sqrt{3 - x}}{x + 5}$$

8. (7pts) Solve and write the solution in interval notation.

$$4x - 3 < 5 \text{ or } 2x + 1 > 10$$

9. (10pts) The diameter of a circle has endpoints $(-1, 2)$ and $(3, -4)$.

a) Find the equation of the circle.

b) Draw the circle in the coordinate plane.

10. (12pts) Two event venues charge the following fees for use of their space.

— Big Enchilada charges a \$210 flat fee that includes two hours, then \$80 per hour after the first two hours.

— Locale Loco charges \$90 per hour.

Assuming your event lasts at least two hours, for which number of hours is Locale Loco a better deal? Solve as an inequality.

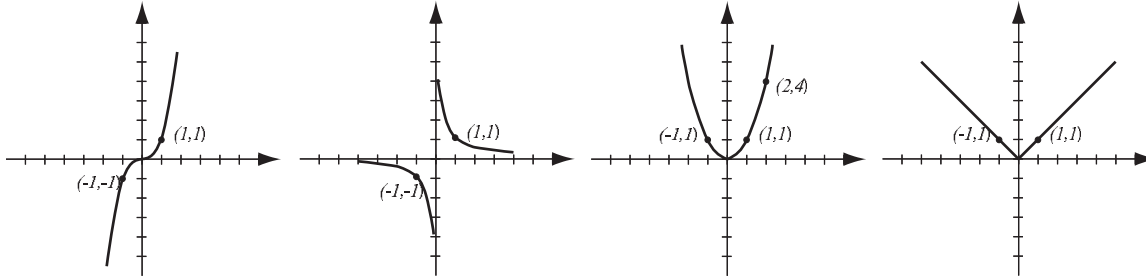
11. (14pts) A truck enters a highway, and a car does the same a quarter of an hour later and drives in the same direction as the truck. Because the car drives 12 miles per hour faster than the truck, it catches up with the truck an hour and a quarter after the car entered.

- a) What are the speeds of the truck and the car?
- b) How far from the entrance of the highway did they meet?

Bonus (10pts) The natural gas bill for a household was \$44.06 in a month when it used 15 hcf of gas. In another month, it used 23 hcf and was billed \$57.02. (Hcf is a unit for quantity of natural gas used.)

- a) Assuming that gas cost $C(x)$ is a linear function of the amount of gas x used (in hcf), write a formula for $C(x)$.
- b) What is the meaning of the slope in this example?

1. (8pts) The following are graphs of basic functions. Write the equation of the graph under each one.



2. (20pts) Let $f(x) = \frac{8}{x-3}$, $g(x) = \frac{2}{x-4}$.

Find the following (simplify where possible):

$$(f + g)(2) =$$

$$(fg)(4) =$$

$$\frac{f}{g}(x) =$$

$$(f \circ g)(5) =$$

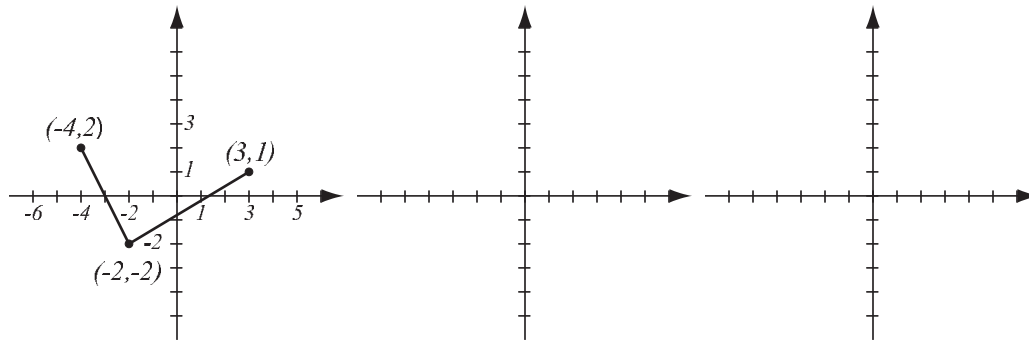
$$(g \circ f)(x) =$$

The domain of $f + g$ in interval notation

3. (6pts) Consider the function $h(x) = \sqrt{\frac{1}{x} + 3}$ and find **two** different solutions to the following problem: find functions f and g so that $h(x) = f(g(x))$, where neither f nor g are the identity function.

4. (6pts) Write the equation for the function whose graph has the following characteristics:
 a) shape of $y = |x|$, shifted up 1 unit.
 b) shape of $y = \sqrt{x}$, stretched horizontally by factor 2, then reflected over the x -axis.

5. (10pts) The graph of $f(x)$ is drawn below. Find the graphs of $2f(x + 1)$ and $-f(-x)$ and label all the relevant points.



6. (8pts) Sketch the graph of the piecewise-defined function:

$$f(x) = \begin{cases} x + 2, & \text{if } -3 \leq x < 1 \\ -2x + 3, & \text{if } 1 \leq x \leq 4 \end{cases}$$

7. (8pts) Find the values of the piecewise-defined function.

$$f(x) = \begin{cases} \sqrt{x} + 2, & \text{if } -4 < x < 2 \\ 3x - 1, & \text{if } 2 \leq x \leq 3 \\ x^2 - 3x, & \text{if } 3 < x \leq 7 \end{cases}$$

$$f(2.5) =$$

$$f(0) =$$

$$f(-5) =$$

$$f(2 \cdot 3) =$$

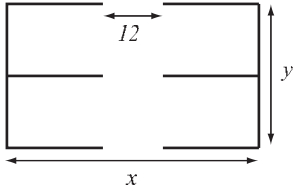
8. (20pts) Let $f(x) = x^3 - 8x$ (answer with 6 decimal points accuracy).

- Use your graphing calculator to accurately draw the graph of f (on paper!). Indicate units on the axes.
- Determine algebraically whether the function is odd, even, or neither.
- Verify your conclusion from b) by stating symmetry.
- Find the local maxima and minima for this function. If there is symmetry, use it to reduce the work here.
- State the intervals where the function is increasing and where it is decreasing.

9. (14pts) A horse breeder wishes to build a stable that is to have area 1200 square feet and four stalls with a 12-ft passageway going through the middle. To minimize cost, the total length of walls has to be as small as possible.

a) Express the total length of walls of the stable as a function of the length of one of the sides x . What is the domain of this function?

b) Graph the function in order to find the minimum. What are the dimensions of the stable that has the smallest total wall length? What is the smallest total wall length?



Bonus. (10pts) Let $f(x) = \frac{2}{x-1}$ and $g(x) = \frac{2+x}{x}$. Find the functions $(f \circ g)(x)$ and $(g \circ f)(x)$.

College Algebra — Exam 3
MAT 140C, Spring 2023 — D. Ivanšić

Name: _____
Show all your work!

Simplify, so that the answer is in form $a + bi$.

1. (5pts) $2i(3i - 2) + 3i(5 + i) =$

2. (5pts) $\frac{4 + i}{3 - 2i} =$

3. (4pts) Simplify and justify your answer.

$i^{99} =$

4. (6pts) Solve the equation by completing the square.

$x^2 - 12x + 4 = 0$

5. (6pts) Solve the inequality. Write the solution in interval form.

$|x - 5| < 3$

6. (6pts) Let $P(x)$ be a polynomial of degree 4.

a) Draw a graph of P that has the maximal number of x -intercepts.

b) Draw a graph of P that has the minimal number of turning points.

7. (12pts) The quadratic function $f(x) = x^2 + 8x$ is given. Do the following without using the calculator.

- a) Find the x - and y -intercepts of its graph, if any.
- b) Find the vertex of the graph.
- c) Sketch the graph of the function.

Solve the equations:

8. (8pts) $\frac{x}{x+4} - \frac{2}{x+1} = \frac{5x-1}{x^2+5x+4}$

9. (8pts) $\sqrt{4x+17} - x = 3$

10. (14pts) The polynomial $f(x) = (x + 2)^2(x - 3)$ is given.

a) What is the end behavior of the polynomial?

b) List all the zeros and their multiplicities. Find the y -intercept.

c) Use the graphing calculator along with a) and b) to accurately sketch the graph of f (yes, on paper!).

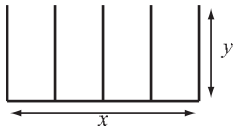
d) Find all the turning points (i.e., local maxima and minima).

11. (12pts) In a rectangle whose area is 5 ft^2 , the length is 2 ft more than the width. What are the dimensions of the rectangle?

12. (14pts) Spiffy Dude is building a quick-oil-change shop with four bays. It has budgeted enough money to build 200 feet of walls, and its goal is to maximize the total area of the shop.

a) Express the total area of the shop as a function of the length of one of the sides. What is the domain of this function?

b) Graph the function in order to find the maximum (no need for the graphing calculator — you should already know what the graph looks like). What are the dimensions of the shop that has the biggest possible total area, and what is the biggest possible total area?



Bonus. (10pts) Write the general solutions of the quadratic equation $ax^2 + bx + c = 0$ using the quadratic formula. Then multiply the two solutions to find a formula for their product.

College Algebra — Exam 4
MAT 140C, Spring 2023 — D. Ivanšić

Name: _____
Show all your work!

1. (8pts) Evaluate without using the calculator. For each problem, write the question you should ask yourself in order to find the logarithms.

$$\log_3 27 = \qquad \log_2 \frac{1}{8} = \qquad \log_a \sqrt[3]{a^4} = \qquad \log_{\sqrt{a}} a^6 =$$

2. (4pts) Use the change-of-base formula and your calculator to find $\log_4 54$ with accuracy 6 decimal places. Show how you obtained your number.

3. (5pts) If $\log_a 3 = 1.098$ and $\log_a 5 = 1.609$, calculate the following values:

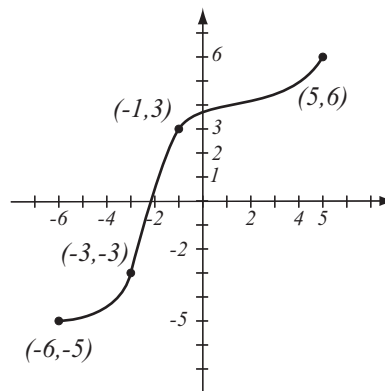
$$\log_a 15 = \qquad \log_a \frac{25}{3} =$$

4. (4pts) Simplify.

$$\ln e^{|x|-1} = \qquad 7^{\log_7 1331} =$$

5. (8pts) If you deposit \$2500 in an account bearing 4.1% interest, compounded quarterly, how much is in the account after 4 years?

6. (6pts) The graph of a function f is given.
- Is this function one-to-one? Justify.
 - If the function is one-to-one, find the graph of f^{-1} , labeling the relevant points, and showing any asymptotes.



7. (9pts) Let $f(x) = \frac{x}{x+3}$.
- Find the formula for f^{-1} .
 - Find the range of f .

8. (6pts) Using transformations, draw the graph of $f(x) = -\ln(x+4)$. Explain how you transform the graph of a basic function in order to get the graph of f . Indicate at least one point on the graph and any asymptotes.

9. (12pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$$\ln(e^2 x^3 \sqrt{y}) =$$

$$\log_3 \frac{9x^2 y^4}{xy^6} =$$

10. (12pts) Write as a single logarithm. Simplify if possible.

$$3 \log(w^3 z^2) + 2 \log(w^2 z^{-4}) =$$

$$4 \log_2(x + 5) + 3 \log_2(x - 1) - 2 \log_2(x^2 + 4x - 5) =$$

Solve the equations.

11. (6pts) $16^{2x+1} = 4^{x+3}$

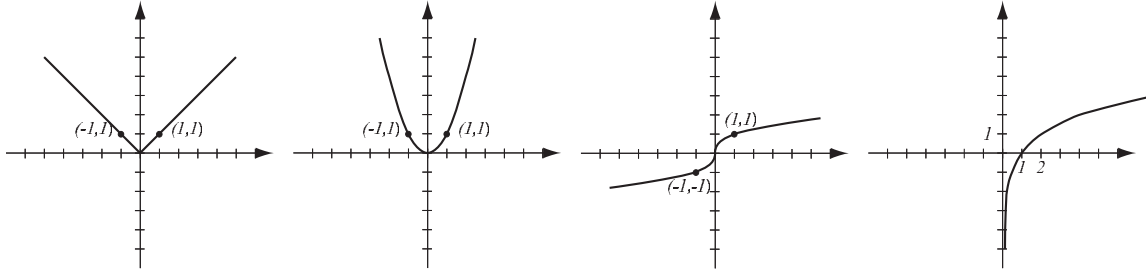
12. (8pts) $4^x = 7^{1-2x}$

- 13.** (12pts) According to census data, the population of Lexington, KY, was 296,000 in 2010 and 323,000 in 2020. Assume that it has grown according to the formula $P(t) = P_0e^{kt}$.
- Find k and write the function that describes the population at time t years since 2010. Graph it on paper.
 - Find the predicted population in the year 2030.

Bonus (10pts) Solve the equation.

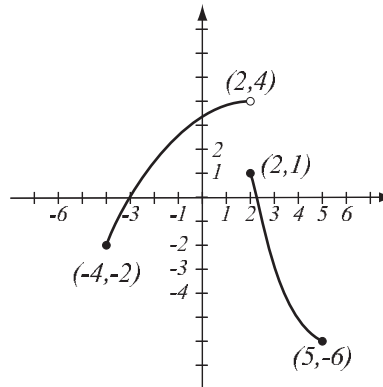
$$\log_2(x - 3) + \log_2(x + 1) = 5$$

1. (8pts) The following are graphs of basic functions. Write the equation of the graph under each one.



2. (8pts) Use the graph of the function f at right to answer the following questions.

- Find: $f(-4) =$ $f(2) =$
- What is the domain of f ?
- What is the range of f ?
- What are the solutions of the equation $f(x) = -2$?



3. (10pts)

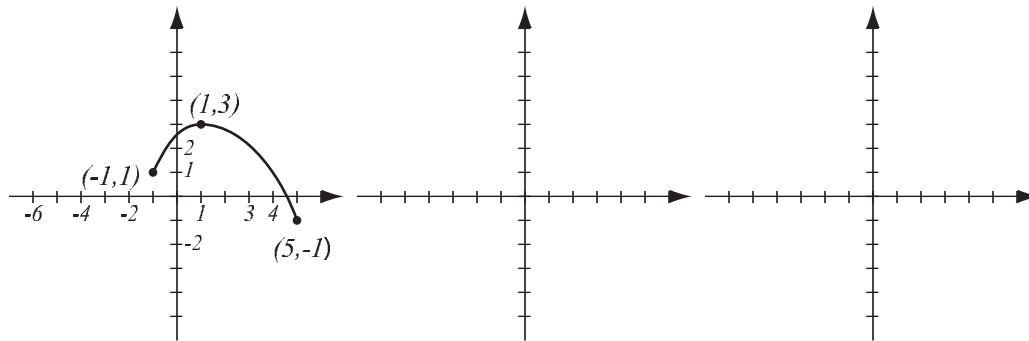
- Find the equation of the line that passes through points $(1, -1)$ and $(4, 5)$.
- Find the equation of the line (in form $y = mx + b$) that is parallel to the line in a) and passes through the point $(-2, 0)$.
- Draw both lines.

4. (3pts) Find the domain of the function $f(x) = \sqrt{2x - 5}$ and write it in interval notation.

5. (6pts) Solve and write the solution in interval notation.

$$|x + 8| \leq 2$$

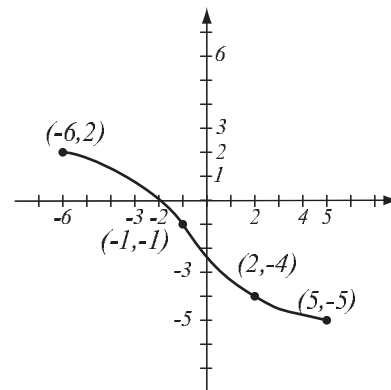
6. (10pts) The graph of $f(x)$ is drawn below. Find the graphs of $f(-x) + 2$ and $2f(x + 3)$ and label all the relevant points.



7. (6pts) The graph of a function f is given.

a) Is this function one-to-one? Justify.

b) If the function is one-to-one, find the graph of f^{-1} , labeling the relevant points.



8. (12pts) The quadratic function $f(x) = x^2 + 4x + 7$ is given. Do the following without using the calculator.

a) Find the x - and y -intercepts of its graph, if any.

b) Find the vertex of the graph.

c) Sketch the graph of the function.

9. (5pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$$\log_5 (125x^7 \sqrt[3]{y}) =$$

10. (6pts) Write as a single logarithm. Simplify if possible.

$$2 \log(x^4 y^2) - 4 \log(x^3 y) =$$

11. (8pts) Let $f(x) = \frac{x}{x^2 - 5}$, $g(x) = \sqrt{x - 3}$. Find the following (simplify where possible):

$$\frac{f}{g}(x) =$$

$$(f \circ g)(x) =$$

12. (20pts) The polynomial $P(x) = x^3 - 4x$ is given (answer with 6 decimals accuracy).

- What is the end behavior of the polynomial?
- Factor the polynomial to find all the zeros and their multiplicities. Find the y -intercept.
- Determine algebraically whether the function is odd, even, or neither.
- Use the graphing calculator along with a) and b) to sketch the graph of P (yes, on paper!).
- Verify your conclusion from c) by stating symmetry.
- Find all the turning points (i.e., local maxima and minima).

13. (8pts) Solve the equation.

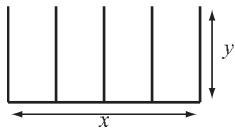
$$\frac{2x}{x+4} + \frac{10x-8}{x^2+2x-8} = \frac{x}{x-2}$$

14. (14pts) A truck enters a highway, and a car does the same a half an hour later and drives in the same direction as the truck. Because the car drives 17 miles per hour faster than the truck, it catches up with the truck an hour and a half after the car entered.

- a) What are the speeds of the truck and the car?
- b) How far from the entrance of the highway did they meet?

15. (14pts) Spiffy Dude is building a quick-oil-change shop with four bays. It has budgeted enough money to build 240 feet of walls, and its goal is to maximize the total area of the shop.

- a) Express the total area of the shop as a function of the length of one of the sides. What is the domain of this function?
- b) Graph the function in order to find the maximum (no need for the graphing calculator — you should already know what the graph looks like). What are the dimensions of the shop that has the biggest possible total area, and what is the biggest possible total area?



- 16.** (12pts) According to census data, the population of Lexington, KY, was 296,000 in 2010 and 323,000 in 2020. Assume that it has grown according to the formula $P(t) = P_0e^{kt}$.
- Find k and write the function that describes the population at time t years since 2010. Graph it on paper.
 - Find the predicted population in the year 2027.

Bonus (10pts) The natural gas bill for a household was \$54.76 in a month when it used 18 hcf of gas. In another month, it used 29 hcf and was billed \$74.67. (Hcf is a unit for quantity of natural gas used.)

- Assuming that gas cost $C(x)$ is a linear function of the amount of gas x used (in hcf), write a formula for $C(x)$.
- What is the meaning of the slope in this example?