College Algebra — Final Exam MAT 140C, Spring 2023 — D. Ivanšić

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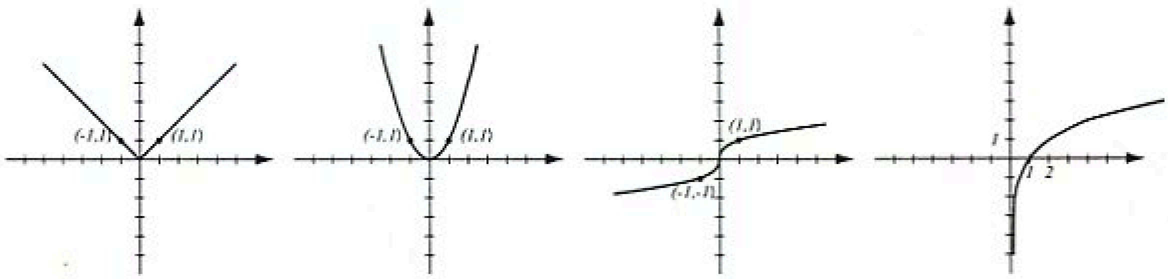
Show all your work!

1. (8pts) The following are graphs of basic functions. Write the equation of the graph under each one.



$y=|x|$ $\quad$ $y=x^2$ $\quad$ $y=\sqrt[3]{x}$ $\quad$ $y=\log_a x,\ a>1$

2. (8pts) Use the graph of the function f at right to answer the following questions.

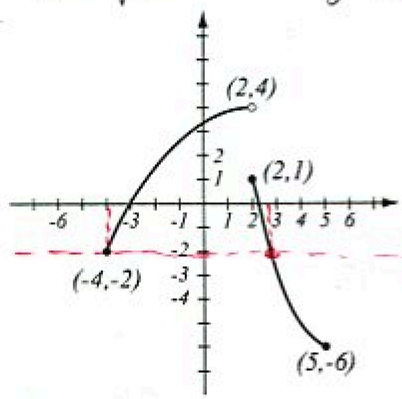
a) Find: $f(-4)=-2$ $\quad f(2)=1$

b) What is the domain of f? $[-4,5]$

c) What is the range of f? $[-6,4)$

d) What are the solutions of the equation $f(x)=-2$?

$x=-4,\ 2.7$



3. (10pts)

a) Find the equation of the line that passes through points $(1,-1)$ and $(4,5)$.

b) Find the equation of the line (in form $y=mx+b$) that is parallel to the line in a) and and passes through the point $(-2,0)$.

c) Draw both lines.

a) $m=\dfrac{5-(-1)}{4-1}=\dfrac{6}{3}=2$

$y-(-1)=2(x-1)$

$y+1=2x-2$

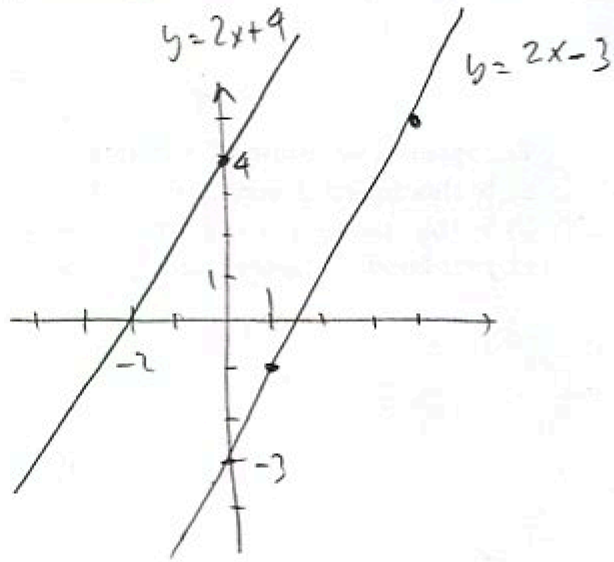
$y=2x-3$

b) slope $=2$, passes through $(-2,0)$

$y-0=2(x-(-2))$

$y=2x+4$

c)



4. (3pts) Find the domain of the function $f(x) = \sqrt{2x-5}$ and write it in interval notation.

Must have $2x-5 \geq 0$

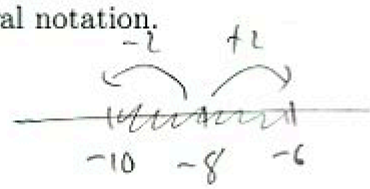
$2x \geq 5$

$x \geq \frac{5}{2}$

$[\frac{5}{2}, \infty)$

5. (6pts) Solve and write the solution in interval notation.

$|x+8| \leq 2$

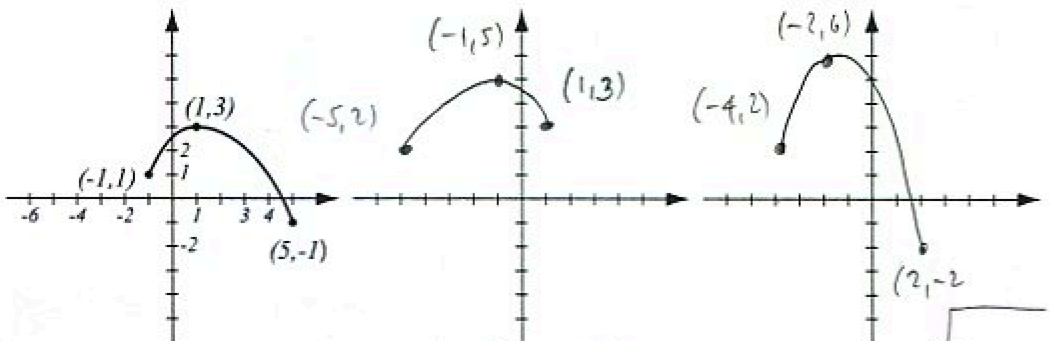


$|x-(-8)| \leq 2$

distance from x to $-8 \leq 2$

$[-10, -6]$

6. (10pts) The graph of $f(x)$ is drawn below. Find the graphs of $f(-x)+2$ and $2f(x+3)$ and label all the relevant points.



vert. stretch, factor=2

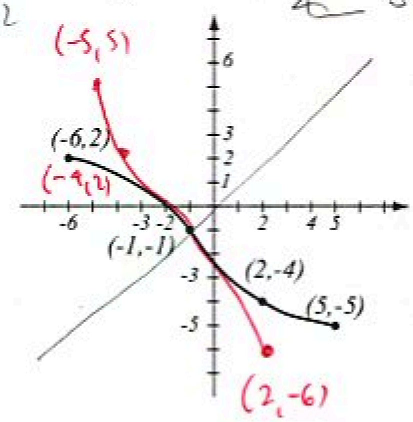
$y = f(-x)+2$

reflect in y-axis shift up 2

$y = 2f(x+3)$

shift left 3

7. (6pts) The graph of a function f is given.
a) Is this function one-to-one? Justify.
b) If the function is one-to-one, find the graph of f^{-1}, labeling the relevant points.



a) It is one-to-one because it passes the horizontal line test

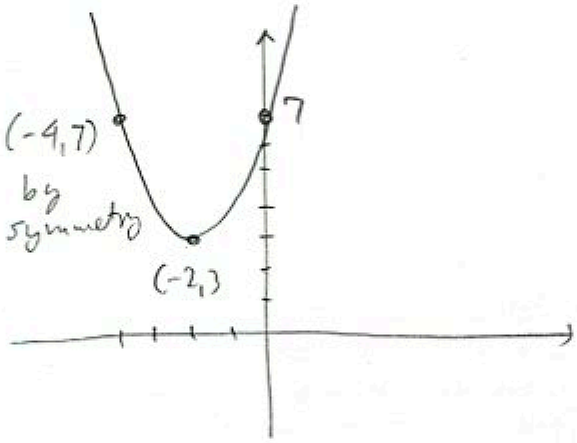
8. (12pts) The quadratic function $f(x) = x^2 + 4x + 7$ is given. Do the following without using the calculator.
a) Find the x- and y-intercepts of its graph, if any.
b) Find the vertex of the graph.
c) Sketch the graph of the function.

a) y-int: $f(0) = 7$

x-int: $x^2 + 4x + 7 = 0$

$$x = \frac{-4 \pm \sqrt{4^2 - 4 \cdot 1 \cdot 7}}{2 \cdot 1}$$

$$= \frac{-4 \pm \sqrt{-12}}{2}$$ no real sol. so no x-int.



b) $h = -\frac{b}{2a} = -\frac{4}{2 \cdot 1} = -2$

$k = f(-2) = 2^2 + 4(-2) + 7 = 3$

9. (5pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$\log_5 (125x^7 \sqrt[3]{y}) = \log_5 125 + \log_5 x^7 + \log_5 y^{\frac{1}{3}}$

$= 3 + 7\log_5 x + \frac{1}{3} \log_5 y$

10. (6pts) Write as a single logarithm. Simplify if possible.

$2\log(x^4y^2) - 4\log(x^3y) = \log (x^4y^2)^2 - \log (x^3y)^4$

$= \log \frac{(x^4y^2)^2}{(x^3y)^4} = \log \frac{x^8y^4}{x^{12}y^4} = \log x^{-4} = \log \frac{1}{x^4}$

11. (8pts) Let $f(x) = \frac{x}{x^2-5}$, $g(x) = \sqrt{x-3}$. Find the following (simplify where possible):

$\frac{f}{g}(x) = \frac{f(x)}{g(x)} = \frac{\frac{x}{x^2-5}}{\sqrt{x-3}} = \frac{x}{x^2-5} \cdot \frac{1}{\sqrt{x-3}}$

$= \frac{x}{(x^2-5)\sqrt{x-3}}$

$(f \circ g)(x) = f(g(x)) = f(\sqrt{x-3})$

$= \frac{\sqrt{x-3}}{\sqrt{x-3}^2 - 5} = \frac{\sqrt{x-3}}{x-8}$

$x-3-5$

12. (20pts) The polynomial $P(x) = x^3 - 4x$ is given (answer with 6 decimals accuracy).
a) What is the end behavior of the polynomial?
b) Factor the polynomial to find all the zeros and their multiplicities. Find the y-intercept.
c) Determine algebraically whether the function is odd, even, or neither.
d) Use the graphing calculator along with a) and b) to sketch the graph of P (yes, on paper!).
e) Verify your conclusion from c) by stating symmetry.
f) Find all the turning points (i.e., local maxima and minima).

a) like x^3,

b) $x^3 - 4x = x(x^2-4) = x(x-2)(x+2)$

y-int: $P(0) = 0$

zeros

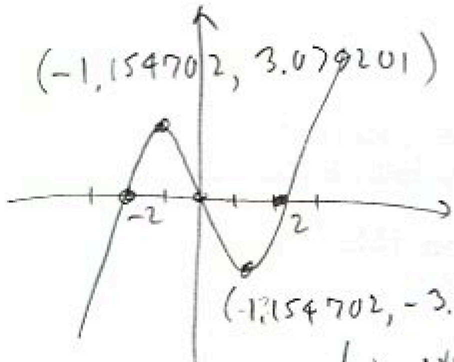
zero	0	2	-2
mult.	1	1	1

c) $f(-x) = (-x)^3 - 4(-x)$

$= -x^3 + 4x = -(x^3 - 4x)$

$= -f(x)$

f is odd



d) graph is symmetric wrt origin

e) Local min. value is $-3.079201 = f(1.154702)$

Local max value is $3.079201 = f(-1.154702$

13. (8pts) Solve the equation.

$$\frac{2x}{x+4} + \frac{10x-8}{x^2+2x-8} = \frac{x}{x-2} \quad \Big| \cdot (x+4)(x-2)$$

$(x+4)(x-2)$

$\frac{2x}{\cancel{x+4}}\cancel{(x+4)}(x-2) + \frac{10x-8}{\cancel{(x+4)(x-2)}}\cancel{(x+4)}\cancel{(x-2)} = \frac{x}{\cancel{x-2}}(x+4)\cancel{(x-2)}$

$2x(x-2) + 10x - 8 = x(x+4)$

$2x^2 - 4x + 10x - 8 = x^2 + 4x$

$x^2 + 2x - 8 = 0$

$(x+4)(x-2) = 0$

$x = -4, 2$

Both give 0 in denominator

so $\boxed{\text{NO SOLUTION}}$

14. (14pts) A truck enters a highway, and a car does the same a half an hour later and drives in the same direction as the truck. Because the car drives 17 miles per hour faster than the truck, it catches up with the truck an hour and a half after the car entered.
a) What are the speeds of the truck and the car?
b) How far from the entrance of the highway did they meet?

	dist.	rate	time
truck	d,	r,	2 hrs
car	d,	$r+17$,	1.5 hrs

$d = r \cdot 2$

$d = (r+17) \cdot 1.5$

$2r = 1.5r + 17 \cdot 1.5$

$0.5r = 17 \cdot 1.5 \quad | \cdot 2$

$2 \cdot 0.5r = 17 \cdot 1.5 \cdot 2$

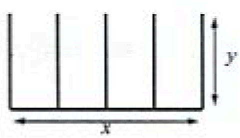
$r = 17 \cdot 3 = 51$

Truck drives 51 mph

Car drives $51 + 17 = 68$ mph

b) $d = 51 \cdot 2 = 102$ miles

15. (14pts) Spiffy Dude is building a quick-oil-change shop with four bays. It has budgeted enough money to build 240 feet of walls, and its goal is to maximize the total area of the shop.
a) Express the total area of the shop as a function of the length of one of the sides. What is the domain of this function?
b) Graph the function in order to find the maximum (no need for the graphing calculator — you should already know what the graph looks like). What are the dimensions of the shop that has the biggest possible total area, and what is the biggest possible total area?



$A = x \cdot y = (240 - 5y)y = -5y^2 + 240y = A(y)$

$x + 5y = 240$

$x = 240 - 5y$

Domain:

Must have

$x \geq 0$

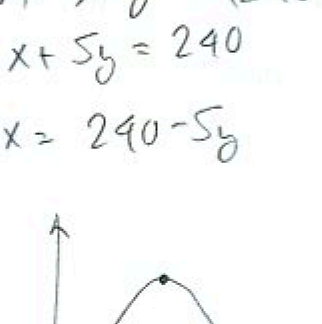
$y \geq 0$

$240 - 5y \geq 0$

$240 \geq 5y$

$48 \geq y$

Domain $[0, 48]$



$h = -\frac{b}{2a} = -\frac{240}{2 \cdot (-5)} = \frac{240}{10} = 24$

$k = (240 - 5 \cdot 24) \cdot 24 = 120 \cdot 24 = 2880$

$x = 240 - 5 \cdot 24$

Dimensions: 120×24

Max. area 2880 ft^2

16. (12pts) According to census data, the population of Lexington, KY, was 296,000 in 2010 and 323,000 in 2020. Assume that it has grown according to the formula $P(t) = P_0 e^{kt}$.
a) Find k and write the function that describes the population at time t years since 2010. Graph it on paper.
b) Find the predicted population in the year 2027.

a) $P(t) = 296\, e^{kt}$

$323 = P(10) = 296\, e^{k \cdot 10}$

$323 = 296\, e^{10k}$

$\frac{323}{296} = e^{10k} \quad | \ln$

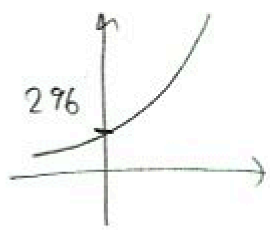
$\ln \frac{323}{296} = 10k$

$k = \frac{\ln \frac{323}{296}}{10} = 0.00872929$

b) 2027 is 17 years after start

$P(17) = 296 \cdot e^{0.008\ldots \cdot 17} = 343.352402$

About 343,352 people



Bonus (10pts) The natural gas bill for a household was \$54.76 in a month when it used 18 hcf of gas. In another month, it used 29 hcf and was billed \$74.67. (Hcf is a unit for quantity of natural gas used.)
a) Assuming that gas cost $C(x)$ is a linear function of the amount of gas x used (in hcf), write a formula for $C(x)$.
b) What is the meaning of the slope in this example?

a) Need line through $(18, 54.76)$ and $(29, 74.67)$

$m = \frac{74.67 - 54.76}{29 - 18} = \frac{19.91}{11} = 1.81$

$y - 54.76 = 1.81(x - 18)$

$y = 1.81x - 32.58 + 54.76$

$= 1.81x + 22.18$

$C(x) = 1.81x + 22.18$

$m = 1.81$ is the cost per hcf of gas