College Algebra — Exam 4
MAT 140C, Spring 2023 — D. Ivanšić

Name: Saul Ocean

Show all your work!

1. (8pts) Evaluate without using the calculator. For each problem, write the question you should ask yourself in order to find the logarithms.

$\log_3 27 = 3$ $\quad 3^? = 27$

$\log_2 \frac{1}{8} = -3$ $\quad 2^? = \frac{1}{8} = \frac{1}{2^3} = 2^{-3}$

$\log_a \sqrt[3]{a^4} = \frac{4}{3}$ $\quad \log_a \sqrt[3]{a^4} = \log_a a^{\frac{4}{3}}$

$\log_{\sqrt{a}} a^6 = 12$ $\quad (\sqrt{a})^? = a^6 \quad (a^{\frac{1}{2}})^? = a^6$

2. (4pts) Use the change-of-base formula and your calculator to find $\log_4 54$ with accuracy 6 decimal places. Show how you obtained your number.

$\log_4 54 = \frac{\log 54}{\log 4} = 2.877444$

3. (5pts) If $\log_a 3 = 1.098$ and $\log_a 5 = 1.609$, calculate the following values:

$\log_a 15 = \log_a (3 \cdot 5)$
$= \log_a 3 + \log_a 5$
$= 1.098 + 1.609$
$= 2.707$

$\log_a \frac{25}{3} = \log_a 25 - \log_a 3$
$= \log_a 5^2 - \log_a 3$
$= 2 \log_a 5 - \log_a 3$
$= 2 \cdot 1.609 - 1.098 = 2.12$

4. (4pts) Simplify.

$\ln e^{|x|-1} = |x| - 1$

$7^{\log_7 1331} = 1331$

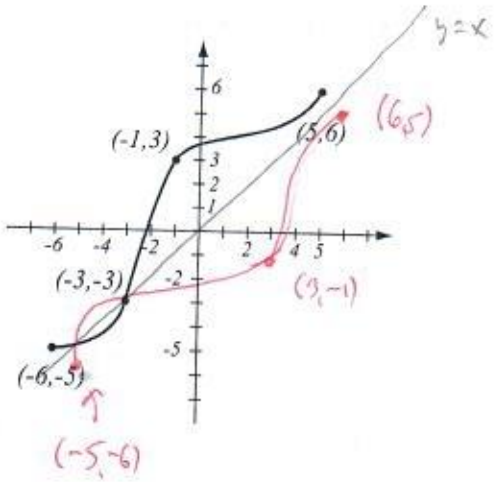
5. (8pts) If you deposit \$2500 in an account bearing 4.1% interest, compounded quarterly, how much is in the account after 4 years?

$A = P\left(1 + \frac{r}{n}\right)^{nt} = 2500\left(1 + \frac{0.041}{4}\right)^{4 \cdot 4}$

$= 2500 \cdot 1.177\ldots$

$= 2943.08$

6. (6pts) The graph of a function f is given.
a) Is this function one-to-one? Justify.
b) If the function is one-to-one, find the graph of f^{-1}, labeling the relevant points, and showing any asymptotes.



a) Yes, it passes the horizontal line test

7. (9pts) Let $f(x) = \dfrac{x}{x+3}$.
a) Find the formula for f^{-1}.
b) Find the range of f.

a) $y = \dfrac{x}{x+3} \quad | \cdot (x+3)$

$y(x+3) = x$

$yx + 3y = x$

$3y = x - yx$

$3y = x(1-y)$

$x = \dfrac{3y}{1-y} \qquad f^{-1}(y) = \dfrac{3y}{1-y}$

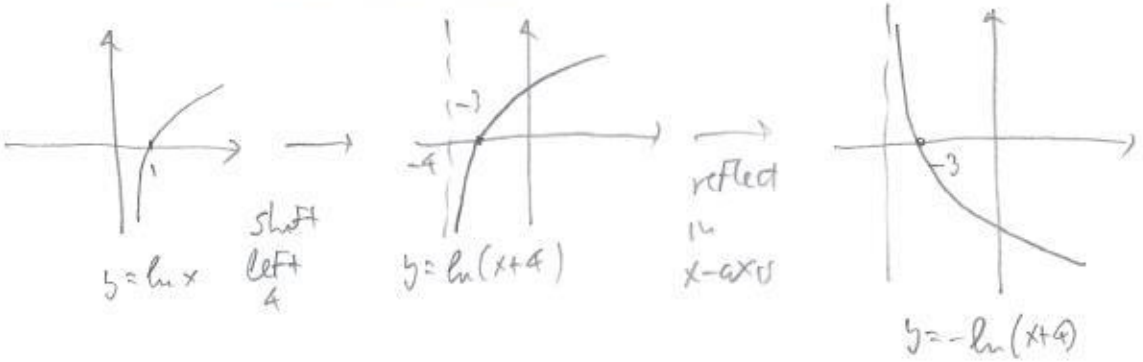
b) Range f = domain f^{-1}

Can't have $1-y=0$

$y=1$

Domain: $(-\infty, 1) \cup (1, \infty)$

8. (6pts) Using transformations, draw the graph of $f(x) = -\ln(x+4)$. Explain how you transform the graph of a basic function in order to get the graph of f. Indicate at least one point on the graph and any asymptotes.



9. (12pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$\ln\left(e^2x^3\sqrt{y}\right) = \ln e^2 + \ln x^3 + \ln y^{\frac{1}{2}}$

$= 2 + 3\ln x + \frac{1}{2}\ln y$

$\log_3 \frac{9x^2y^4}{xy^6} = \log_3 9 + \log_3 x^2 + \log_3 y^4 - \log_3 x - \log_3 y^6$

$= 2 + 2\log_3 x + 4\log_3 y - \log_3 x - 6\log_3 y$

$= 2 + \log_3 x - 2\log_3 y$

10. (12pts) Write as a single logarithm. Simplify if possible.

$3\log(w^3z^2) + 2\log(w^2z^{-4}) = \log(w^3z^2)^3 + \log(w^2z^{-4})^2$

$= \log\left((w^3z^2)^3(w^2z^{-4})^2\right)$

$= \log(w^9z^6w^4z^{-8}) = \log(w^{13}z^{-2}) = \log\frac{w^{13}}{z^2}$

$4\log_2(x+5) + 3\log_2(x-1) - 2\log_2(x^2+4x-5) = \log_2(x+5)^4 + \log_2(x-1)^3 - \log_2(x^2+4x-5)^2$

$= \log_2 \frac{(x+5)^4(x-1)^3}{((x+5)(x-1))^2} = \log_2 \frac{(x+5)^4(x-1)^3}{(x+5)^2(x-1)^2} = \log_2\left((x+5)^2(x-1)\right)$

$(x+5)(x-1)$

Solve the equations.

11. (6pts) $16^{2x+1} = 4^{x+3}$

$(4^2)^{2x+1} = 4^{x+3}$

$4^{4x+2} = 4^{x+3}$

$4x+2 = x+3$

$3x = 1$

$x = \frac{1}{3}$

12. (8pts) $4^x = 7^{1-2x}$ | $\ln$

$\ln 4^x = \ln 7^{1-2x}$

$x\ln 4 = (1-2x)\ln 7$

$x\ln 4 = \ln 7 - 2x\ln 7$

$x\ln 4 + 2x\ln 7 = \ln 7$

$x(\ln 4 + 2\ln 7) = \ln 7$

$x = \frac{\ln 7}{\ln 4 + 2\ln 7} = 0{,}368675$

13. (12pts) According to census data, the population of Lexington, KY, was 296,000 in 2010 and 323,000 in 2020. Assume that it has grown according to the formula $P(t) = P_0 e^{kt}$.
a) Find k and write the function that describes the population at time t years since 2010. Graph it on paper.
b) Find the predicted population in the year 2030.

a) $P(t) = 296e^{kt}$

$323 = P(10) = 296e^{k \cdot 10}$

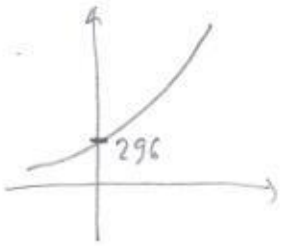
$323 = 296e^{k \cdot 10}$

$\frac{323}{296} = e^{k \cdot 10}$ | ln

$\ln \frac{323}{296} = \ln e^{k \cdot 10}$

$\ln \frac{323}{296} = k \cdot 10$

$k = \frac{\ln \frac{323}{296}}{10} = 0.00872929$



$P(t) = 296e^{0.00872929t}$

b) $P(20) = 296e^{0.00872929 \cdot 20}$

$= 352,962838$

About 352,000 people.

Bonus (10pts) Solve the equation.

$\log_2(x-3) + \log_2(x+1) = 5$

$\log_2((x-3)(x+1)) = 5$

$2^{\log_2(x-3)(x+1)} = 2^5$

$(x-3)(x+1) = 32$

$x^2 - 2x - 3 = 32$

$x^2 - 2x - 35 = 0$

$(x-7)(x+5) = 0$

$x = 7, -5$

$\log_2(-5-3) + \log_2(-5-1) = 5$

-5 gives negative number inside log

$\log_2(7-3) + \log_2(7+1) = 5$

7 is ok

$x = 7$ is the solution