College Algebra — Exam 3
MAT 140C, Spring 2023 — D. Ivanšić

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Show all your work!

Simplify, so that the answer is in form $a+bi$.

1. (5pts) $2i(3i-2)+3i(5+i) = 6i^2 - 4i + 15i + 3i^2$

$= -6 + 11i - 3 = -9 + 11i$

2. (5pts) $\dfrac{4+i}{3-2i} = \dfrac{4+i}{3-2i} \cdot \dfrac{3+2i}{3+2i} = \dfrac{12+8i+3i+2i^2}{3^2-(2i)^2} = \dfrac{12+11i-2}{9-(-4)}$

$= \dfrac{10+11i}{13} = \dfrac{10}{13} + \dfrac{4}{13}i$

3. (4pts) Simplify and justify your answer.

$i^{99} = i^{96} \cdot i^3 = 1 \cdot (-i) = -i$

$(i^4)^{24}$

4. (6pts) Solve the equation by completing the square.

$x^2 - 12x + 4 = 0 \quad | + 6^2$

$x^2 - 2\cdot x \cdot 6 + 6^2 + 4 = 6^2$

$(x-6)^2 = 36 - 4$

$(x-6)^2 = 32$

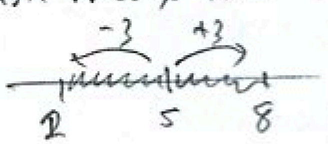
$x - 6 = \pm\sqrt{32}$

$x = 6 \pm \sqrt{\underset{16\cdot 2}{32}} = 6 \pm 4\sqrt{2}$

5. (6pts) Solve the inequality. Write the solution in interval form.

$|x-5| < 3$

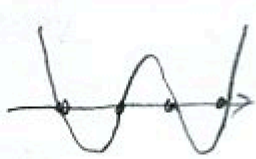
dist. from x to $5 < 3$



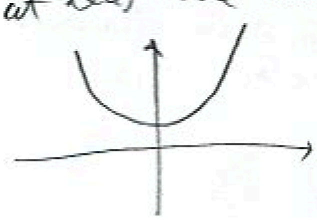
$(2, 8)$

6. (6pts) Let $P(x)$ be a polynomial of degree 4.
a) Draw a graph of P that has the maximal number of x-intercepts.
b) Draw a graph of P that has the minimal number of turning points.

a) Needs to have 4 x-int



b) Due to U-shape, has to have at least one turning point.



7. (12pts) The quadratic function $f(x) = x^2 + 8x$ is given. Do the following without using the calculator.

a) Find the x- and y-intercepts of its graph, if any.

b) Find the vertex of the graph.

c) Sketch the graph of the function.

a) x-int: $x^2+8x=0$

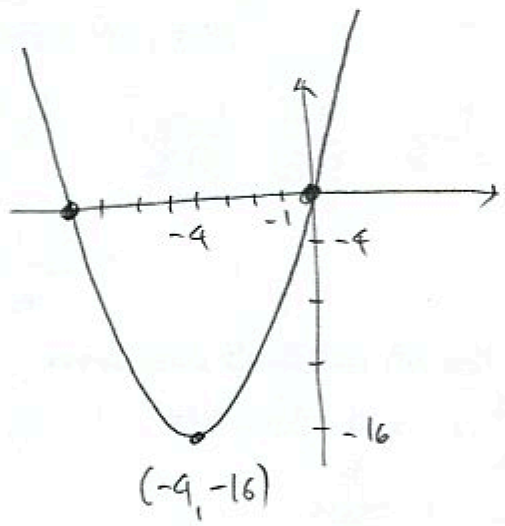
$x(x+8)=0$

$x=0,-8$

y-int: $f(0)=0$

b) $h=-\frac{b}{2a} = -\frac{8}{2\cdot 1} = -4$

$k = f(-4) = (-4)^2 - 32 = -16$



Solve the equations:

8. (8pts) $\frac{x}{x+4} - \frac{2}{x+1} = \frac{5x-1}{x^2+5x+4} \quad \Big| \cdot (x+1)(x+4)$

$(x+1)(x+4)$

$\frac{x}{\cancel{x+4}}(x+1)\cancel{(x+4)} - \frac{2}{\cancel{x+1}}\cancel{(x+1)}(x+4) = \frac{5x-1}{\cancel{(x+1)(x+4)}}\cancel{(x+1)(x+4)}$

$x(x+1) - 2(x+4) = 5x-1$

$x^2 + x - 2x - 8 = 5x - 1 \quad \big| -5x+1$

$x^2 - 6x - 7 = 0$

$(x-7)(x+1) = 0$

$x = 7, \cancel{-1}$

-1 gives 0 in denominator, so not a sol.

$x = 7$

9. (8pts) $\sqrt{4x+17} - x = 3$

$\sqrt{4x+17} = 3 + x \quad \big|^2$

$\sqrt{4x+17}^2 = 3^2 + 2\cdot 3 \cdot x + x^2$

$4x + 17 = 9 + 6x + x^2 \quad \big| -4x-17$

$0 = x^2 + 2x - 8$

$(x+4)(x-2) = 0$

$x = \cancel{-4}, 2$ $\quad x=2$ only solution

$\sqrt{4(-4)+17} - (-4) \stackrel{?}{=} 3$

$\sqrt{1} + 4 \stackrel{?}{=} 3$ no

$\sqrt{4\cdot 2+17} - 2 = 3$

$\sqrt{25} - 2 = 3$ yes

10. (14pts) The polynomial $f(x) = (x+2)^2(x-3)$ is given.
a) What is the end behavior of the polynomial?
b) List all the zeros and their multiplicities. Find the y-intercept.
c) Use the graphing calculator along with a) and b) to accurately sketch the graph of f (yes, on paper!).
d) Find all the turning points (i.e., local maxima and minima).

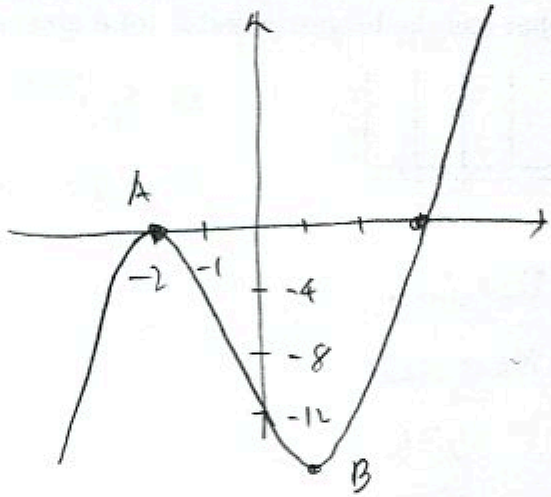
a) like $(x)^2(x) = x^3$

b)

zero	-2	3
mult	2	1

y-int: $f(0) = 2^2(-3) = -12$

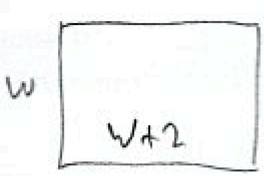
c)



Turning points: $A = (-2, 0)$

$B = (-1.333334, -18.51852)$

11. (12pts) In a rectangle whose area is $5\,\text{ft}^2$, the length is 2 ft more than the width. What are the dimensions of the rectangle?



$w(w+2) = 5$

$w^2 + 2w = 5$

$w^2 + 2w - 5 = 0$

$$w = \frac{-2 \pm \sqrt{2^2 - 4\cdot 1\cdot(-5)}}{2\cdot 1}$$

$$= \frac{-2 \pm \sqrt{24}}{2} = \frac{-2 \pm 2\sqrt{6}}{2} = \frac{2(-1 \pm \sqrt{6})}{2} = -1 \pm \sqrt{6}$$

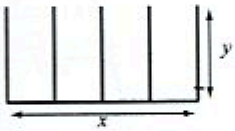
Since $w \geq 0$, $-1-\sqrt{6}$ cannot be a solution,

thus $w = -1+\sqrt{6}$

12. (14pts) Spiffy Dude is building a quick-oil-change shop with four bays. It has budgeted enough money to build 200 feet of walls, and its goal is to maximize the total area of the shop.

a) Express the total area of the shop as a function of the length of one of the sides. What is the domain of this function?

b) Graph the function in order to find the maximum (no need for the graphing calculator — you should already know what the graph looks like). What are the dimensions of the shop that has the biggest possible total area, and what is the biggest possible total area?

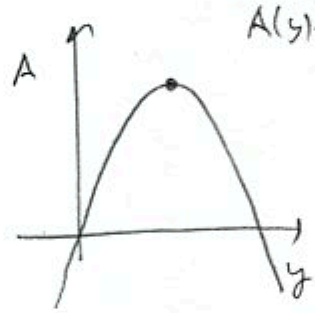


$x + 5y = 200$

$x = 200 - 5y$

$A = xy = (200-5y)y = -5y^2 + 200y$

$A(y) = -5y^2 + 200y$



Domain:

Must have:

$y \geq 0$

$x \geq 0$

$200 - 5y \geq 0$

$200 \geq 5y$

$40 \geq y$

Domain: $[0, 40]$

$h = -\frac{b}{2a} = -\frac{200}{2(-5)} = \frac{200}{10} = 20$

$k = A(20) = -5 \cdot 20^2 + 200 \cdot 20 = 2000$

$(x = 200 - 5 \cdot 20)$

Dimensions: $\overset{x}{100} \times \overset{y}{20}$

Max area: 2000 ft^2

Bonus. (10pts) Write the general solutions of the quadratic equation $ax^2 + bx + c = 0$ using the quadratic formula. Then multiply the two solutions to find a formula for their product.

Product of solutions is $\dfrac{\overbrace{-b + \sqrt{b^2-4ac}}^{(A + B)}}{2a} \cdot \dfrac{\overbrace{-b - \sqrt{b^2-4ac}}^{(A - B)}}{2a}$

$= \dfrac{\overset{A^2}{(-b)^2} - \overset{B^2}{\sqrt{b^2-4ac}^{\,2}}}{4a^2} = \dfrac{b^2 - (b^2 - 4ac)}{4a^2} = \dfrac{\cancel{4}a c}{\cancel{4a^2}} = \dfrac{c}{a}$