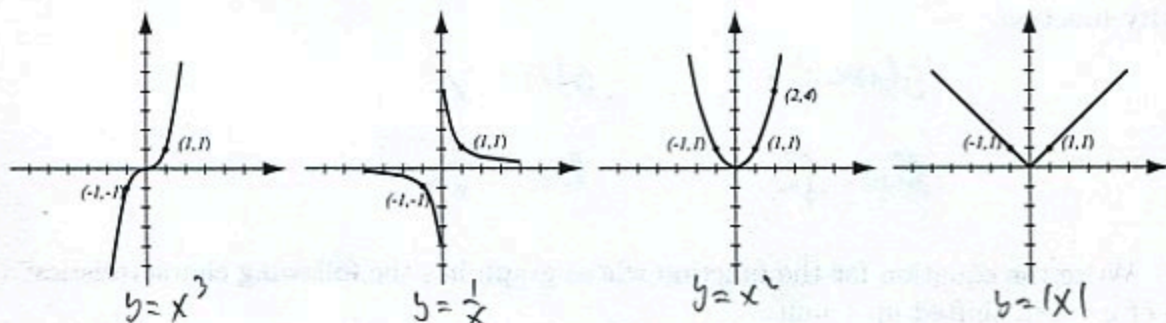


1. (8pts) The following are graphs of basic functions. Write the equation of the graph under each one.



2. (20pts) Let $f(x) = \frac{8}{x-3}$, $g(x) = \frac{2}{x-4}$.

Find the following (simplify where possible):

$$\begin{aligned} (f+g)(2) &= f(2) + g(2) \\ &= \frac{8}{2-3} + \frac{2}{2-4} = \frac{8}{-1} + \frac{2}{-2} = -8 - 1 = -9 \end{aligned}$$

$$\begin{aligned} \frac{f}{g}(x) &= \frac{f(x)}{g(x)} = \frac{\frac{8}{x-3}}{\frac{2}{x-4}} = \frac{8}{x-3} \cdot \frac{x-4}{2} \\ &= \frac{4(x-4)}{x-3} \end{aligned}$$

$$(fg)(4) = f(4) \cdot g(4) = \frac{8}{4-3} \cdot \frac{2}{4-4}$$

undefined
 $\frac{2}{0}$ not def

$$\begin{aligned} (f \circ g)(5) &= f(g(5)) = f\left(\frac{2}{5-4}\right) = f(2) \\ &= \frac{8}{2-3} = \frac{8}{-1} = -8 \end{aligned}$$

$$(g \circ f)(x) = g(f(x)) = g\left(\frac{8}{x-3}\right) = \frac{2}{\frac{8}{x-3} - 4} = \frac{x-3}{8 - 4(x-3)} = \frac{2(x-3)}{8 - 4x + 12} = \frac{2(x-3)}{20 - 4x}$$

The domain of $f+g$ in interval notation

Domain of f : Can't have: $x-3=0$
 $x=3$

Domain of g : Can't have $x-4=0$
 $x=4$

~~_____~~ f
~~_____~~ g
~~_____~~ \leftarrow
 3 4
 overlap

Domain of $f+g$ is

$$(-\infty, 3) \cup (3, 4) \cup (4, \infty)$$

3. (6pts) Consider the function $h(x) = \sqrt{\frac{1}{x} + 3}$ and find two different solutions to the following problem: find functions f and g so that $h(x) = f(g(x))$, where neither f nor g are the identity function.

$$g(x) = \frac{1}{x} + 3$$

$$g(x) = \frac{1}{x}$$

$$f(x) = \sqrt{x}$$

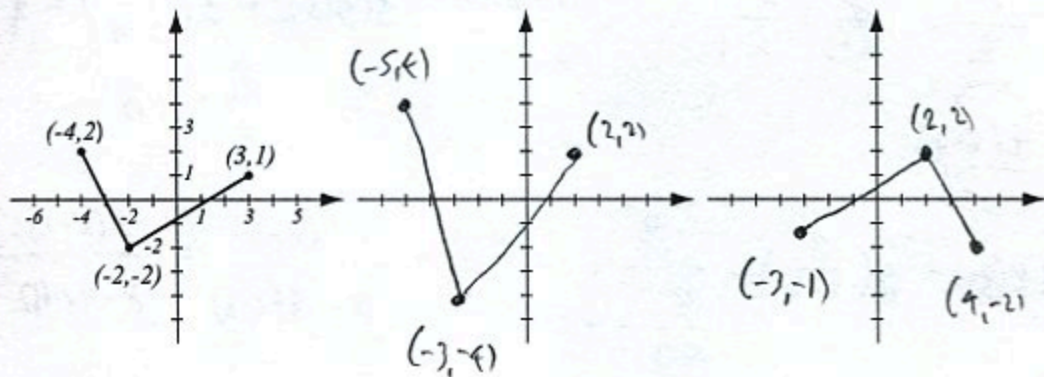
$$f(x) = \sqrt{x+3}$$

4. (6pts) Write the equation for the function whose graph has the following characteristics:
 a) shape of $y = |x|$, shifted up 1 unit.
 b) shape of $y = \sqrt{x}$, stretched horizontally by factor 2, then reflected over the x -axis.

$$a) y = |x| \rightsquigarrow y = |x| + 1$$

$$b) y = \sqrt{x} \rightsquigarrow y = \sqrt{\frac{1}{2}x} \rightsquigarrow y = -\sqrt{\frac{1}{2}x}$$

5. (10pts) The graph of $f(x)$ is drawn below. Find the graphs of $2f(x+1)$ and $-f(-x)$ and label all the relevant points.



$2f(x+1)$
 \leftarrow left 1
 \uparrow stretch vertically,
 factor = 2

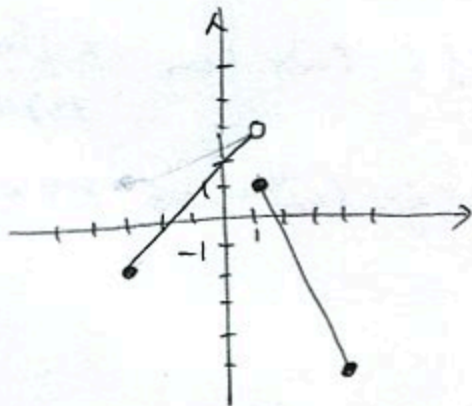
$-f(-x)$
 \leftarrow reflect in y -axis
 \uparrow reflect in x -axis

6. (8pts) Sketch the graph of the piecewise-defined function:

$$f(x) = \begin{cases} x+2, & \text{if } -3 \leq x < 1 \\ -2x+3, & \text{if } 1 \leq x \leq 4 \end{cases}$$

x	$x+2$
-3	-1
1	3

x	$-2x+3$
1	1
4	-5



7. (8pts) Find the values of the piecewise-defined function.

$$f(x) = \begin{cases} \sqrt{x} + 2, & \text{if } -4 < x < 2 \\ 3x - 1, & \text{if } 2 \leq x \leq 3 \\ x^2 - 3x, & \text{if } 3 < x \leq 7 \end{cases}$$

$$f(2.5) = 3 \cdot 2.5 - 1 = 7.5 - 1 = 6.5$$

$$f(0) = \sqrt{0} + 2 = 2$$

$$f(-5) = \text{not defined}$$

(-5 not in any of given intervals)

$$f(2 \cdot 3) = f(6) = 6^2 - 3 \cdot 6 = 36 - 18 = 18$$

8. (20pts) Let $f(x) = x^3 - 8x$ (answer with 6 decimal points accuracy).

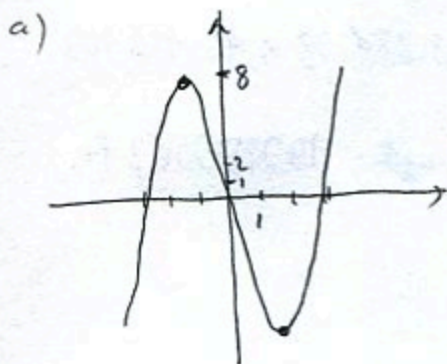
a) Use your graphing calculator to accurately draw the graph of f (on paper!). Indicate units on the axes.

b) Determine algebraically whether the function is odd, even, or neither.

c) Verify your conclusion from b) by stating symmetry.

d) Find the local maxima and minima for this function. If there is symmetry, use it to reduce the work here.

e) State the intervals where the function is increasing and where it is decreasing.



d)

$$\text{Local max is } 8.709297 = f(-1.632993)$$

$$\text{Local min is } -8.709297 = f(1.632993)$$

e)

$$\text{Increasing on } (-\infty, -1.632993) \cup (1.632993, \infty)$$

$$\text{Decreasing on } (-1.632993, 1.632993)$$

b)

$$f(-x) = (-x)^3 - 8(-x)$$

$$= -x^3 + 8x$$

$$= -(x^3 - 8x) = -f(x)$$

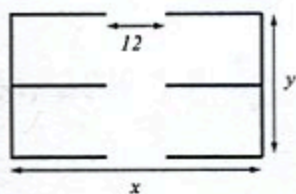
Function is odd

c) Symmetric wrt origin

9. (14pts) A horse breeder wishes to build a stable that is to have area 1200 square feet and four stalls with a 12-ft passageway going through the middle. To minimize cost, the total length of walls has to be as small as possible.

a) Express the total length of walls of the stable as a function of the length of one of the sides x . What is the domain of this function?

b) Graph the function in order to find the minimum. What are the dimensions of the stable that has the smallest total wall length? What is the smallest total wall length?



$$l = 3(x-12) + 2y = 3x + 2y - 36 = 3x + 2 \cdot \frac{1200}{x} - 36$$

$$xy = 1200$$

$$l(x) = 3x + \frac{2400}{x} - 36$$

$$y = \frac{1200}{x}$$

Domain:

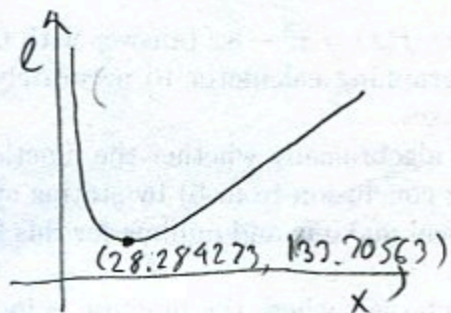
Must have $x \geq 12$

$$y > 0$$

$$\frac{1200}{x} > 0$$

holds for every x

$$\text{Domain} = [12, \infty)$$



Dimensions: $28.284273 \times 42.426403$

Smallest wall length: 133.70563 ft

Bonus. (10pts) Let $f(x) = \frac{2}{x-1}$ and $g(x) = \frac{2+x}{x}$. Find the functions $(f \circ g)(x)$ and $(g \circ f)(x)$.

$$(f \circ g)(x) = f(g(x)) = f\left(\frac{2+x}{x}\right) = \frac{2}{\frac{2+x}{x}-1} \cdot \frac{x}{x} = \frac{2x}{2+x-x} = \frac{2x}{2} = x$$

$$(g \circ f)(x) = g(f(x)) = g\left(\frac{2}{x-1}\right) = \frac{2 + \frac{2}{x-1}}{\frac{2}{x-1}} \cdot \frac{x-1}{x-1} = \frac{2(x-1)+2}{2} = \frac{2x-2+2}{2} = \frac{2x}{2} = x$$