1. (8pts) Use the graph of the function f at right to answer the following questions.
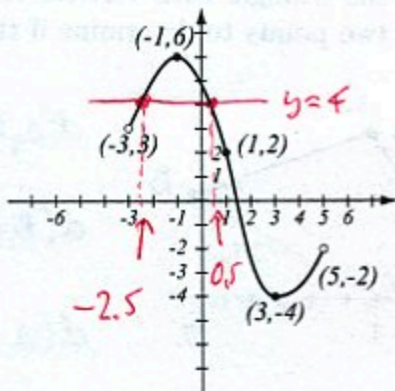
a) Find: $f(1) = 2$ $f(-5) =$ not defined

b) What is the domain of f? $(-3, 5)$

c) What is the range of f? $[-4, 6]$

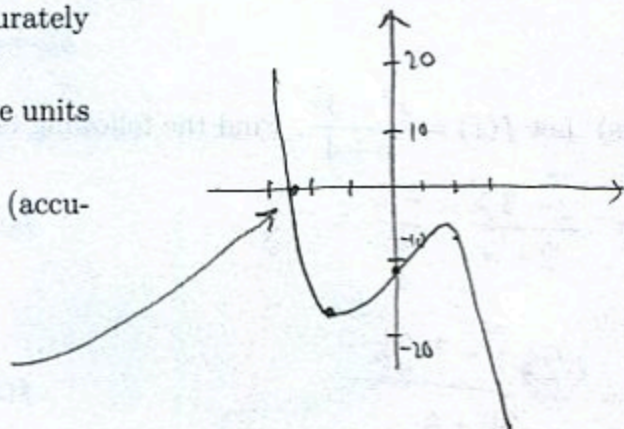d) What are the solutions of the equation $f(x) = 4$? $x = -2.5, 0.5$

2. (10pts) Use your calculator to accurately sketch the graph of $y = -x^3 + 8x - 11$.

a) Draw the graph on paper and indicate units on the axes.

b) Find all the $x-$ and $y-$intercepts (accuracy: 6 decimal points).
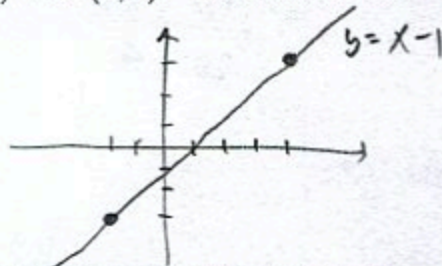
$y-$int: $x = 0$, $y = -11$

$x-$int: $x = -3.357947$

3. (5pts) Draw the line that passes through points $(-2, -3)$ and $(4, 3)$. Then write the equation of the line in form $y = mx + b$.

$m = \dfrac{3-(-3)}{4-(-2)} = \dfrac{6}{6} = 1$

Eq. of line:

$y - 3 = 1 \cdot (x - 4)$

$y = x - 1$

4. (10pts) Find the equation of the line (in form $y = mx + b$) that is perpendicular to the line $2x - 3y = 9$ and passes through the point $(2, 1)$. Draw both lines.

$2x - 3y = 9$
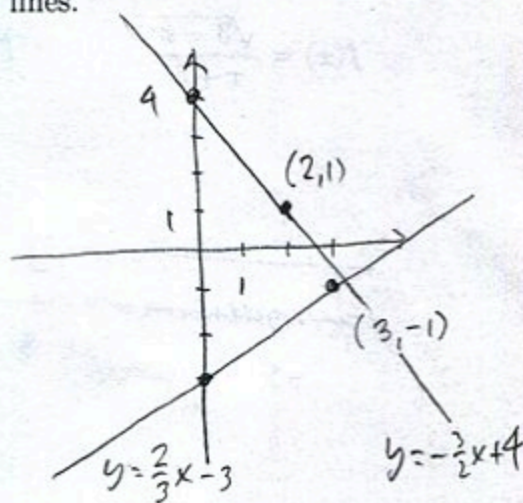
$-3y = -2x + 9 \ | \div (-3)$

$y = \frac{2}{3}x - 3$

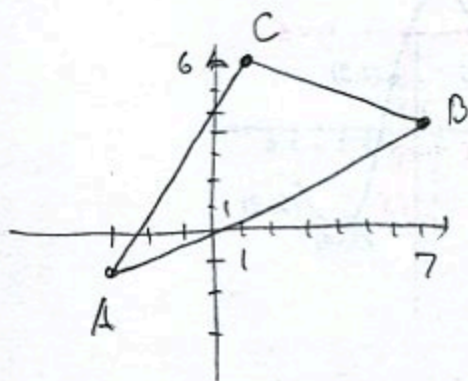Slope of this line is $\frac{2}{3}$,

slope of perp. line is $-\frac{3}{2}$

$y - 1 = -\frac{3}{2}(x - 2)$

$y - 1 = -\frac{3}{2}x + 3$

$\boxed{y = -\frac{3}{2}x + 4}$

5. (8pts) Draw the triangle with vertices $A = (-3, -1)$, $B = (7, 4)$ and $C = (1, 7)$. Use distance between two points to determine if this is a right triangle.

$$d(A,B) = \sqrt{(7-(-3))^2 + (4-(-1))^2} = \sqrt{100 + 25} = \sqrt{125}$$

$$d(B,C) = \sqrt{(1-7)^2 + (7-4)^2} = \sqrt{36 + 9} = \sqrt{45}$$

$$d(A,C) = \sqrt{(1-(-3))^2 + (7-(-1))^2} = \sqrt{16 + 64} = \sqrt{80}$$

$$\sqrt{80}^2 + \sqrt{45}^2 \overset{?}{=} \sqrt{125}^2$$

$$80 + 45 = 125 \quad \text{true, so triangle is right}$$

6. (9pts) Let $f(x) = \dfrac{x^2 - 3x}{x+4}$. Find the following (simplify where appropriate).

$$f(2) = \frac{2^2 - 3\cdot 2}{2+4} = \frac{-2}{6} = -\frac{1}{3}$$

$$f(-4) = \frac{(-4)^2 - 3(-4)}{-4+4} = \frac{28}{0} \quad \text{not defined}$$

$$f(2b) = \frac{(2b)^2 - 3\cdot 2b}{2b+4}$$

$$= \frac{4b^2 - 6b}{2b+4}$$

$$f(x+1) = \frac{(x+1)^2 - 3(x+1)}{x+1+4}$$

$$= \frac{x^2 + 2x + 1 - 3x - 3}{x+5}$$

$$= \frac{x^2 - x - 2}{x+5}$$

7. (7pts) Find the domain of the function below and write it using interval notation.

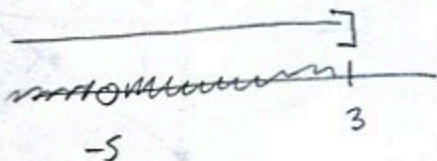$$f(x) = \frac{\sqrt{3-x}}{x+5}$$

Must have:

$3 - x \geq 0$

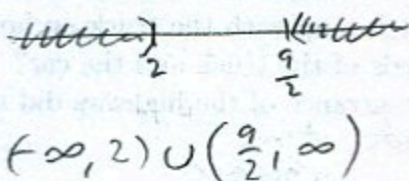$3 \geq x$

$x \leq 3$

Can't have

$x + 5 = 0$

$x = -5$

$(-\infty, -5) \cup (-5, 3]$

8. (7pts) Solve and write the solution in interval notation.

$4x - 3 < 5$ or $2x + 1 > 10$

$4x < 8 \qquad\qquad 2x > 9$

$x < 2$ or $\quad x > \dfrac{9}{2} \qquad\qquad (-\infty, 2) \cup \left(\dfrac{9}{2}, \infty\right)$

9. (10pts) The diameter of a circle has endpoints $(-1, 2)$ and $(3, -4)$.
a) Find the equation of the circle.
b) Draw the circle in the coordinate plane.

a) Center is midpoint of given points

$$C = \left(\frac{-1+3}{2}, \frac{2+(-4)}{2}\right) = \left(\frac{2}{2}, \frac{-2}{2}\right) = (1, -1)$$
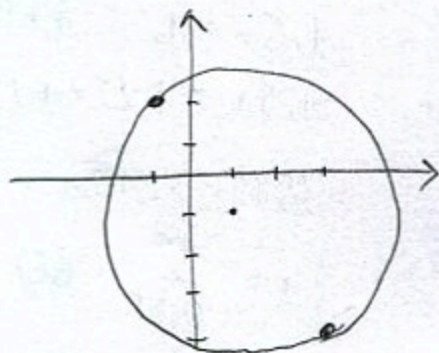
radius = distance from center to a point
on circle $\qquad (1, -1) \qquad (-1, 2)$

$$r = \sqrt{(-1-1)^2 + (2-(-1))^2} = \sqrt{4+9} = \sqrt{13}$$

Equation of circle:

$$(x-1)^2 + (y-(-1))^2 = \sqrt{13}^2$$

$$(x-1)^2 + (y+1)^2 = 13$$

10. (12pts) Two event venues charge the following fees for use of their space.
— Big Enchilada charges a $210 flat fee that includes two hours, then $80 per hour after the first two hours.
— Locale Loco charges $90 per hour.
Assuming your event lasts at least two hours, for which number of hours is Locale Loco a better deal? Solve as an inequality.

Let x = number of hours needed

B.E. cost: $\quad 210 + 80(x-2)$

L.L. cost: $\quad 90x$

For events lasting up to 5 hours, Locale Loco is the better deal.

Wish to know for which x is

$$L.L. \leq B.E.$$

$$90x \leq 210 + 80(x-2)$$

$$90x \leq 210 + 80x - 160 \quad |-80x$$

$$10x \leq 50 \quad\quad |\div 10$$

$$x \leq 5$$

11. (14pts) A truck enters a highway, and a car does the same a quarter of an hour later and drives in the same direction as the truck. Because the car drives 12 miles per hour faster than the truck, it catches up with the truck an hour and a quarter after the car entered.
a) What are the speeds of the truck and the car?
b) How far from the entrance of the highway did they meet?

dist, rate, time

$$\underrightarrow{\begin{array}{ccc} d & r & 1.25+0.25 \end{array}}\ \text{truck}$$

$$\underrightarrow{\begin{array}{ccc} d & r+12 & 1.25 \end{array}}\ \text{car}$$

a) truck speed: 60 mph

car speed: 72 mph

$d = r \cdot 1.5$

$d = (r+12) 1.25$

b) $d = 60 \cdot 1.5 = 90$ miles

$1.5r = 1.25(r+12)$

$1.5r = 1.25r + 15 \quad |-1.25r$

$0.25r = 15$

$r = \dfrac{15}{0.25} = 60$

Bonus (10pts) The natural gas bill for a household was \$44.06 in a month when it used 15 hcf of gas. In another month, it used 23 hcf and was billed \$57.02. (Hcf is a unit for quantity of natural gas used.)
a) Assuming that gas cost $C(x)$ is a linear function of the amount of gas x used (in hcf), write a formula for $C(x)$.
b) What is the meaning of the slope in this example?

Need line through $(15, 44.06)$
$(23, 57.02)$

b) Slope is cost of gas per hcf

it is \$1.62 per hcf.

$m = \dfrac{57.02 - 44.06}{23 - 15} = \dfrac{12.96}{8} = 1.62$

$y - 44.06 = 1.62(x-15)$

$y = 1.62x - 24.30 + 44.06$

$y = C(x) = 1.62x + 19.76$