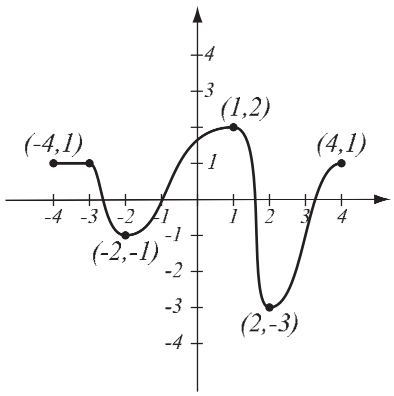
College Algebra — Lecture notes
MAT 140, Spring 2023 — D. Ivanšić

2.1 Increasing and Decreasing Functions

Example. On the graph of the function, observe where it is rising (increasing) and falling (decreasing).



f is increasing on

f is decreasing on

f is constant on

is a local (relative) maximum

is a local (relative) minimum

Definition.
$f(c)$ is a local maximum if there is an open interval I around c so that $f(x) < f(c)$ for all x in I, $x \neq c$.
$f(c)$ is a local minimum if there is an open interval I around c so that $f(x) > f(c)$ for all x in I, $x \neq c$.

Example. Sketch the graph of $f(x) = x^3 - 5x^2 + 2x - 5$.
a) Find its local maxima and minima.
b) Find the intervals of increase and decrease.

Piecewise defined functions

Example. Sketch the graph of the function

$$f(x) = \begin{cases} -3x + 6, & \text{if } x < 3 \\ 2x - 7, & \text{if } x \geq 3 \end{cases}$$

Max and Min Applications

Example. Farmer Bill has 300 meters of fencing. He would like to enclose a rectangular plot of land so that its area is the largest possible.

a) Draw three different rectangular enclosures that Bill could make and compute their areas.

b) Express the area of the enclosure as a function of the length of one of the sides x. What is the domain of this function?

c) Graph the function in order to find the maximum. What are the dimensions of the enclosure that has the greatest area?

Example. Farmer Hyacinth wants to fence a rectangular area of 4km^2 and then divide it in half with a fence parallel to a side of the rectangle. She wishes to do this so the cost of the fence is minimal (thus, the length of fence is minimal).
a) Draw three different rectangular enclosures that Hyacinth could make and compute the length of fence used.
b) Express the length of fence used as a function of the length of one of the sides x. What is the domain of this function?
c) Graph the function in order to find the minimum. What are the dimensions of the enclosure that has the smallest length of fence?

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2.2, 2.3 Algebra of Functions, Composition

When we have two functions there are various ways to combine them.

Example. Let $f(x) = \dfrac{1}{x-2}$ and $g(x) = \sqrt{x+4}$.

We can define new functions $f+g$, $f-g$, $f \cdot g$, $\dfrac{f}{g}$.

$(f+g)(6) =$

$(f-g)(6) =$

$(f \cdot g)(6) =$

$\dfrac{f}{g}(6) =$

Or, in general:

$(f+g)(x) =$

$(f-g)(x) =$

$(f \cdot g)(x) =$

$\dfrac{f}{g}(x) =$

Domain of functions $f+g$, $f-g$, $f \cdot g$ is the intersection of domains of f and g.

Find the domains of $f+g$, $f-g$, $f \cdot g$ in this example.

Domain of $\dfrac{f}{g}$ is the intersection of domains of f and g, with any x-values for which $g(x) = 0$ excluded.

Find the domain of $\dfrac{f}{g}$ in this example.

Composition of functions

Example. A car is moving east from an intersection at 35mph. Express the distance of the car to a point P that is 1 mile north of the intersection as a function of time t.

Example. Let $f(x) = \dfrac{1}{x-2}$ and $g(x) = \sqrt{x+4}$.

$1 \overset{g}{\longmapsto} \qquad \overset{f}{\longmapsto}$

$-2 \overset{g}{\longmapsto} \qquad \overset{f}{\longmapsto}$

$x \overset{g}{\longmapsto} \qquad \overset{f}{\longmapsto}$

We got a new function $f(g(x))$, denoted $(f \circ g)(x)$, called the composite of functions f and g.

Example. Find the composites of the functions below.

$f(x) = x^2 - 3x + 5 \qquad g(x) = \dfrac{1}{x} \qquad h(x) = \sqrt{2x-7}$

$(f \circ g)(x) =$

$(g \circ f)(x) =$

$(h \circ g)(x) =$

$(h \circ h)(x) =$

$(g \circ g)(x) =$

$(h \circ f)(x) =$

Example. Consider the functions below and find **two** different solutions to the following problem: find functions f and g so that $f(g(x))$ is the given function.

$\sqrt{x^2 - x + 6}$	$g(x) =$ $f(x) =$	$g(x) =$ $f(x) =$
$\sqrt{x+3} + 12$	$g(x) =$ $f(x) =$	$g(x) =$ $f(x) =$
$\dfrac{6}{2x+3}$	$g(x) =$ $f(x) =$	$g(x) =$ $f(x) =$

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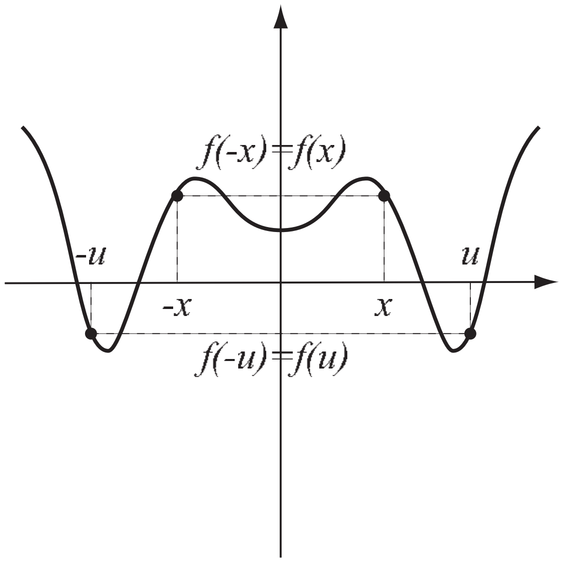
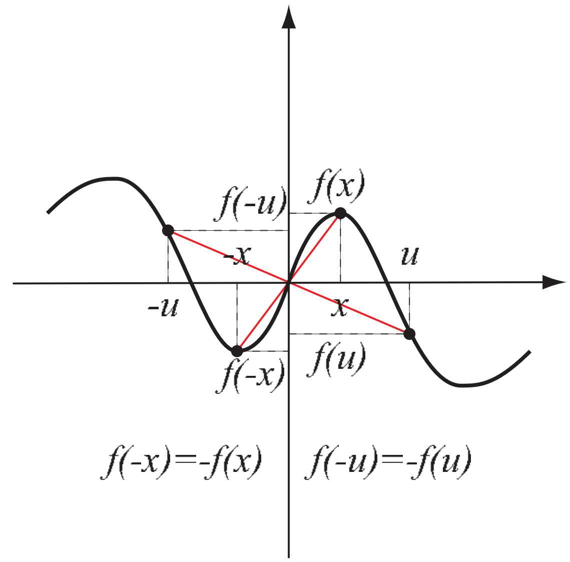
2.4 Even and Odd Functions

Graphs of functions can be symmetric with respect to the y-axis and with respect to the origin. Note they cannot be symmetric with respect to the x axis, because they would fail the vertical line test.

Consider these graphs, symmetric with respect to the y-axis and with respect to the origin.





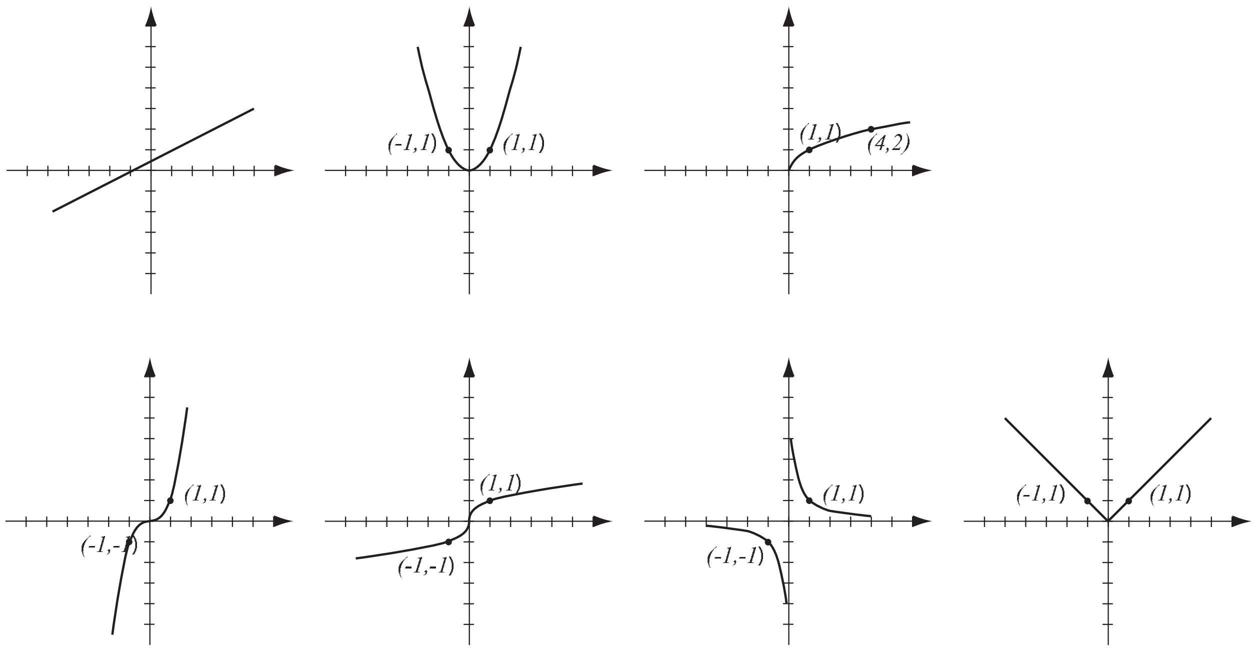
Definition.
A function f is even if $f(-x) = f(x)$ (symmetric with respect to the y-axis).
A function f is odd if $f(-x) = -f(x)$ (symmetric with respect to the origin).

Example. For the following functions:
a) determine algebraically whether they are odd, even, or neither
b) use the calculator to draw their graphs here and verify your conclusions by stating symmetry.

$f(x) = x^4 - 3x^2$ $\qquad$ $g(x) = x^5 - 3x^2 + x$ $\qquad$ $h(x) = x^5 - 5x^3 + 4x$

2.5 Transformations of Graphs

Graphs of basic functions:



In each of the examples below, use your calculator to help you draw the graphs of the functions f, g, and h on paper (all three on one coordinate system). Explain how the three graphs are related, in other words, how to get the graphs of g and h from the graph of f?

Example

$f(x) = x^2$

$g(x) = x^2 + 3 = f(x) + 3$

$h(x) = x^2 - 1 = f(x) - 1$

g, h have form $f(x) + b$

Example

$f(x) = x^3$

$g(x) = (x + 5)^3 = f(x + 5)$

$h(x) = (x - 2)^3 = f(x - 2)$

g, h have form $f(x + d)$

Example

$f(x) = x^3 - 7x^2 + 10x$

$g(x) = -(x^3 - 7x^2 + 10x) = -f(x)$

$h(x) = (-x)^3 - 7(-x)^2 + 10(-x) = f(-x)$

In each of the examples below, use your calculator to help you draw the graphs of the functions f, g, and h on paper (all three on one coordinate system). Pay special attention to x- and y-intercepts and local minima. Explain how the three graphs are related, in other words, how to get the graphs of g and h from the graph of f?

Example

$f(x) = x^2 - 2x - 3$

$g(x) = 2(x^2 - 2x - 3) = 2f(x)$

$h(x) = \frac{1}{2}(x^2 - 2x - 3) = \frac{1}{2}f(x)$

g, h have form $af(x)$

Example

$f(x) = x^2 - 2x - 3$

$g(x) = (2x)^2 - 2(2x) - 3 = f(2x)$

$h(x) = (\frac{1}{2}x)^2 - 2(\frac{1}{2}x) - 3 = f(\frac{1}{2}x)$

g, h have form $f(cx)$

Fill in the following table with your findings from the examples on this sheet.

To get graph of	Do this to graph of f	How coordinates on graph change
$y = f(x) + b$		
$y = f(x + d)$		
$y = -f(x)$		
$y = f(-x)$		
$y = af(x)$, $a > 0$		
$y = f(cx)$, $c > 0$		