Calculus 1 — Exam 5
MAT 250, Fall 2022 — D. Ivanšić

Name: _____

Show all your work!

Find the following antiderivatives or definite integrals.

1. (3pts) $\displaystyle\int \sqrt[5]{x^2}\,dx =$

2. (3pts) $\displaystyle\int \cos\left(3x+\frac{\pi}{2}\right)\,dx =$

3. (6pts) $\displaystyle\int \sqrt{t}(t^2-3t)\,dt =$

4. (4pts) $\displaystyle\int_1^e \frac{1}{x}\,dx =$

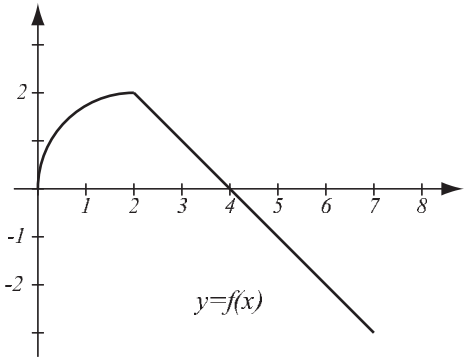
5. (7pts) $\displaystyle\int_0^{\frac{\pi}{6}} \frac{1+\cos^2\theta}{\cos^2\theta}\,d\theta =$

6. (6pts) Find $f(x)$ if $f'(x)=\dfrac{4}{\sqrt{1-x^2}}$ and $f\left(\dfrac{1}{2}\right)=3$.

7. (15pts) The function $f(x) = 4 - x^2$ is given on the interval $[0, 3]$.
a) Write the Riemann sum M_6 for this function with six subintervals, taking sample points to be midpoints. Do not evaluate the expression.
b) Illustrate with a diagram, where appropriate rectangles are clearly visible. What does M_6 represent?

8. (13pts) Find $\int_0^3 x - 1\, dx$ in two ways (they'd better give you the same answer!):
a) Using the "area" interpretation of the integral. Draw a picture.
b) Using the Evaluation Theorem.

9. (10pts) The graph of a function f, consisting of lines and parts of circles, is shown. Evaluate the integrals.

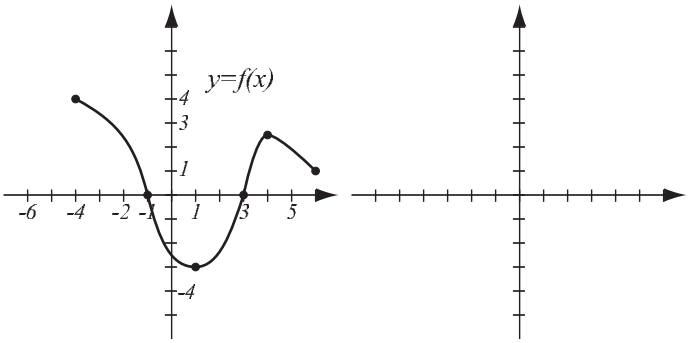


$\int_0^2 f(x)\,dx =$

$\int_2^7 f(x)\,dx =$

$\int_0^7 f(x)\,dx =$

10. (8pts) Pictured is the graph of $f(x)$. Sketch the graph of the antiderivative $F(x)$ of $f(x)$, if it is known that $F(-4) = 0$. Relevant points have been highlighted.



11. (7pts) Use the inequality $m(b-a) \le \int_a^b f(x)\,dx \le M(b-a)$, where $m \le f(x) \le M$ on $[a,b]$, to give an estimate of $\int_{-4}^{-2} f(x)\,dx$, where f is the function pictured in the preceding problem.

12. (6pts) Write using sigma notation:

$$\frac{2}{x^4} + \frac{3}{x^6} + \frac{4}{x^8} + \cdots + \frac{9}{x^{18}} =$$

13. (12pts) Toxic sludge is being deposited into a collection pool at rate $6\sqrt{t} + 3t$ cubic meters per hour.
a) Use the Net Change Theorem to find how much sludge was added from $t = 0$ to $t = 4$ hours.
b) If at time $t = 0$ there were 12 cubic meters of sludge in the pool, how much is there at $t = 4$ hours?

Bonus. (10pts) Recall that $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$. Use this formula to evaluate the sum below. (Note that it does not start with 1, how do you handle this?)

$$\sum_{i=4}^{n} (7 + 2i)$$