Name:
Show all your work!

1. (30pts) Let $f(x) = \frac{x^2}{x^2 + 9}$. Draw an accurate graph of f by following the guidelines.

a) Find the intervals of increase and decrease, and local extremes.

b) Find the intervals of concavity and points of inflection.

c) Find $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$. d) Use information from a)-c) to sketch the graph.

2. (16pts) Let $f(x) = x^2 e^x$. Find the absolute minimum and maximum values of f on the interval [-4, 1].

3. (14pts) The graph of f is given. Use it to draw the graphs of f' and f'' in the coordinate systems provided. Pay attention to increasingness, decreasingness and concavity of f. The relevant special points have been highlighted.



- 4. (18pts) Consider $f(x) = \sin^2 x \cos x$ on the interval $[0, 2\pi]$.
- a) Verify that the function satisfies the assumptions of the Mean Value Theorem.
- b) Find all numbers c that satisfy the conclusion of the Mean Value Theorem.

5. (22pts) A box with a square base uses 50 square inches of material (that is its surface area, including top and bottom). Find the dimensions x, y of the box that give the maximal possible volume of the box.



Bonus. (10pts) Let $f(x) = \frac{5}{9}x^{\frac{9}{5}} - \frac{15}{4}x^{\frac{4}{5}}$. Note domain is all real numbers. a) Find the intervals of increase and decrease, and local extremes.

- b) Find the intervals of concavity and points of inflection.
- c) Use information from a) and b) to sketch the graph.