Calculus 1 — Exam 3
MAT 250, Fall 2022 — D. Ivanšić

Name: _____

Show all your work!

Differentiate and simplify where appropriate:

1. (5pts) $\dfrac{d}{dx}\, e^{x^2-x+3} =$

2. (6pts) $\dfrac{d}{dx}\left(\sqrt{x} - \dfrac{1}{\sqrt{x}}\right) e^x =$

3. (6pts) $\dfrac{d}{dt}\, \dfrac{\arctan t}{t^2} =$

4. (7pts) $\dfrac{d}{dx}\, \ln\left(\dfrac{x+2}{x-2}\right)^3$

5. (7pts) $\dfrac{d}{d\theta}\, \ln(\sec\theta + \tan\theta) =$

6. (9pts) Use logarithmic differentiation to find the derivative of $y = (\sin x)^{\cos x}$.

Find the limits algebraically. Graphs of basic functions will help, as will L'Hospital's rule, where appropriate.

7. (2pts) $\lim_{x\to-\infty} e^{3x} =$

8. (7pts) $\lim_{x\to\infty} \arctan\left(\frac{x^2+5x+1}{x+7}\right) =$

9. (6pts) $\lim_{x\to\infty} \frac{x^2}{2^x} =$

10. (9pts) $\lim_{x\to0} \frac{\tan x - x}{x^3} =$

11. (8pts) $\lim_{x\to0+} x^x =$

12. (12pts) Let $f(x) = \ln x$.
a) Write the linearization of $f(x)$ at $a = 1$.
b) Use the linearization to estimate $\ln 1.2$.
c) In the same coordinate system, draw rough graphs of the function and the linearization and determine if the estimate overshoots or undershoots $\ln 1.2$.

13. (9pts) In a right triangle, the hypothenuse is known to be 5 inches. One of the angles is measured to be $\frac{\pi}{6}$ radians, with maximum error 0.1 radians. Use differentials to estimate the maximum possible error and relative error when computing the length of the side adjacent to the angle.

14. (7pts) Let $f(x) = x^3 - x$. Use the theorem on derivatives of inverses to find $(f^{-1})'(6)$.

Bonus. (10pts) Let $f(x) = x^n$, $x \geq 0$, where n is a positive integer. We have justified the rule for the derivative of f using the definition by computing a limit. Use the derivative of f and either the theorem on derivatives of inverses, or implicit differentiation, to justify the rule for the derivative of $\sqrt[n]{x}$.