## Calculus 1 - Exam 1 MAT 250, Fall 2022 - D. Ivanšić

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1. (16pts) Use the graph of the function to answer the following. Justify your answer if a limit does not exist.
$\lim _{x \rightarrow 5+} f(x)=$
$\lim _{x \rightarrow 5^{-}} f(x)=$
$\lim _{x \rightarrow 5} f(x)=$
$\lim _{x \rightarrow-\infty} f(x)=$
$\lim _{x \rightarrow \infty} f(x)=$
$\lim _{x \rightarrow 3} f(x)=$
List points in domain of $f$ where $f$ is not continuous and justify why it is not continuous at those points.
2. (8pts) Draw the graph of a function $f(x)$ defined on the interval $(1,6)$ which satisfies:

$$
\begin{aligned}
& \lim _{x \rightarrow 6^{-}} f(x)=-\infty \\
& \lim _{x \rightarrow 1^{+}} f(x)=\infty
\end{aligned}
$$

$f$ is discontinuous at $x=3$, continuous elsewhere
the equation $f(x)=4$ has no solution
3. (10pts) Find $\lim _{x \rightarrow \infty} \frac{3+\cos x}{x^{2}}$. Use the theorem that rhymes with a dairy product people often put in sandwiches.

Find the following limits algebraically. Do not use the calculator.
4. $(5 \mathrm{pts}) \lim _{x \rightarrow 5} \frac{x^{2}-25}{x^{2}+2 x-35}=$
5. (7pts) $\lim _{x \rightarrow \infty} \frac{x+2}{x^{2}-3 x+1}=$
6. (6pts) $\lim _{x \rightarrow 3^{-}} \frac{x-4}{6-2 x}=$
7. (7pts) $\lim _{x \rightarrow 4} \frac{x-4}{\sqrt{x}-2}=$
8. $(7 \mathrm{pts}) \lim _{x \rightarrow 0} \frac{\sin (2 x) \sin x}{3 x^{2}}=$
9. (14pts) The equation $2^{x}=5-x$ is given.
a) Use the Intermediate Value Theorem to show it has a solution in the interval $(0,2)$.
b) Use your calculator to find an interval of length at most 0.01 that contains a solution of the equation. Then use the Intermediate Value Theorem to justify why your interval contains the solution.
10. (10pts) Consider the limit $\lim _{x \rightarrow 0}(1-2 x)^{\frac{1}{x}}$. Use your calculator (don't forget parentheses) to estimate this limit with accuracy 3 decimal points. Write a table of values that will support your answer.

| $x$ | $(1-2 x)^{\frac{1}{x}}$ | $x$ | $(1-2 x)^{\frac{1}{x}}$ |
| :--- | :--- | :--- | :--- |
|  |  |  |  |
|  |  |  |  |

11. (10pts) Consider the function defined below.
a) Explain why the function is continuous on intervals $(-\infty, 1)$ and $(1, \infty)$
b) Is the function continuous at point $x=1$ ?
$f(x)= \begin{cases}4 x-5, & \text { if } x \leq 1 \\ x^{2}-2 x, & \text { if } x>1 .\end{cases}$

Bonus. (10pts) Evaluate the function at the given $x$ 's. Then, based on the table, state what $\lim _{x \rightarrow 0} \frac{\left(x^{4}+2\right)^{3}-8}{x^{4}}$ appears to be. Explain any strange numbers you are getting.

| $x$ | $\frac{\left(x^{4}+2\right)^{3}-8}{x^{4}}$ |
| ---: | ---: |
| 0.1 |  |
| 0.01 |  |
| 0.001 |  |
| $10^{-4}$ |  |
| $10^{-5}$ |  |
| $10^{-6}$ |  |

