## Calculus 1 — Exam 1 MAT 250, Fall 2022 — D. Ivanšić

Name:

Show all your work!

**1.** (16pts) Use the graph of the function to answer the following. Justify your answer if a limit does not exist.

$$\lim_{x \to 5^+} f(x) =$$
$$\lim_{x \to 5^-} f(x) =$$
$$\lim_{x \to -\infty} f(x) =$$
$$\lim_{x \to -\infty} f(x) =$$
$$\lim_{x \to \infty} f(x) =$$
$$\lim_{x \to 3} f(x) =$$

List points in domain of f where f is not continuous and justify why it is not continuous at those points.



**2.** (8pts) Draw the graph of a function f(x) defined on the interval (1,6) which satisfies:

$$\lim_{x \to 6^{-}} f(x) = -\infty$$
$$\lim_{x \to 1^{+}} f(x) = \infty$$

f is discontinuous at x = 3, continuous elsewhere

the equation f(x) = 4 has no solution

**3.** (10pts) Find  $\lim_{x\to\infty} \frac{3+\cos x}{x^2}$ . Use the theorem that rhymes with a dairy product people often put in sandwiches.

Find the following limits algebraically. Do not use the calculator.

4. (5pts) 
$$\lim_{x \to 5} \frac{x^2 - 25}{x^2 + 2x - 35} =$$

5. (7pts) 
$$\lim_{x \to \infty} \frac{x+2}{x^2 - 3x + 1} =$$

**6.** (6pts) 
$$\lim_{x \to 3^-} \frac{x-4}{6-2x} =$$

7. (7pts) 
$$\lim_{x \to 4} \frac{x-4}{\sqrt{x-2}} =$$

8. (7pts) 
$$\lim_{x \to 0} \frac{\sin(2x)\sin x}{3x^2} =$$

9. (14pts) The equation  $2^x = 5 - x$  is given.

a) Use the Intermediate Value Theorem to show it has a solution in the interval (0, 2).

b) Use your calculator to find an interval of length at most 0.01 that contains a solution of the equation. Then use the Intermediate Value Theorem to justify why your interval contains the solution.

10. (10pts) Consider the limit  $\lim_{x\to 0} (1-2x)^{\frac{1}{x}}$ . Use your calculator (don't forget parentheses) to estimate this limit with accuracy 3 decimal points. Write a table of values that will support your answer.

$(1-2x)^{\frac{1}{x}}$		x	$(1-2x)^{\frac{1}{x}}$
	_		
	$(1-2x)^{\frac{1}{x}}$	$(1-2x)^{\frac{1}{x}}$	$(1-2x)^{\frac{1}{x}} \qquad x$

11. (10pts) Consider the function defined below. a) Explain why the function is continuous on intervals  $(-\infty, 1)$  and  $(1, \infty)$ b) Is the function continuous at point x = 1?

 $f(x) = \begin{cases} 4x - 5, & \text{if } x \le 1\\ x^2 - 2x, & \text{if } x > 1. \end{cases}$ 

**Bonus.** (10pts) Evaluate the function at the given x's. Then, based on the table, state what  $\lim_{x\to 0} \frac{(x^4+2)^3-8}{x^4}$  appears to be. Explain any strange numbers you are getting.

x	$\frac{(x^4+2)^3-8}{x^4}$
0.1	
0.01	
0.001	
$10^{-4}$	
$10^{-5}$	
$10^{-6}$	