Calculus 1 — Exam 1
MAT 250, Fall 2023 — D. Ivanšić

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Show all your work!

1. (16pts) Use the graph of the function to answer the following. Justify your answer if a limit does not exist.

$\lim_{x\to -4+} f(x) = 1$

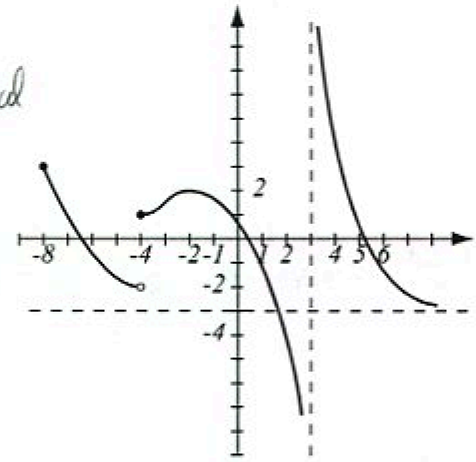
$\lim_{x\to -4-} f(x) = -2$

$\lim_{x\to -4} f(x) =$ DNE, one-sided limits not equal

$\lim_{x\to \infty} f(x) = -3$

$\lim_{x\to 3-} f(x) = -\infty$

$\lim_{x\to 3+} f(x) = \infty$



List points in $[-8, \infty)$ where f is not continuous and justify why it is not continuous at those points.

At $x=-4$, $\lim_{x\to -4} f(x)$ does not exist, at $x=3$, $f(3)$ not defined

2. (6pts) Let $\lim_{x\to 1} f(x) = 4$ and $\lim_{x\to 1} g(x) = -3$. Use limit laws to find the limit below and show each step.

$$\lim_{x\to 1} \frac{x^3 + g(x)^2}{2+\sqrt{f(x)}} = \frac{\lim_{x\to 1}(x^3+g(x)^2)}{\lim_{x\to 1}(2+\sqrt{f(x)})} = \frac{\lim_{x\to 1} x^3 + \lim_{x\to 1} g(x)^2}{\lim_{x\to 1} 2 + \lim_{x\to 1}\sqrt{f(x)}} = \frac{(\lim_{x\to 1} x)^3 + (\lim_{x\to 1} g(x))^2}{2+\sqrt{\lim_{x\to 1} f(x)}}$$

$$= \frac{1^3+(-3)^2}{2+\sqrt{4}} = \frac{10}{4} = \frac{5}{2}$$

3. (10pts) Find $\lim_{x\to 0+} \sqrt{x}\left(3+\sin\frac{1}{x}\right)$. Use the theorem that rhymes with a vegetable that looks like small green balls.

$-1 \le \sin\frac{1}{x} \le 1$

$2 \le 3+\sin\frac{1}{x} \le 4$

$2\sqrt{x} \le \sqrt{x}\left(3+\sin\frac{1}{x}\right) \le 4\sqrt{x}$

$\left.\begin{array}{l}\lim_{x\to 0+} 2\sqrt{x} = 2\cdot 0 = 0 \\ \lim_{x\to 0+} 4\sqrt{x} = 4\cdot 0 = 0\end{array}\right\}$ are equal, so

by squeeze theorem

$\lim_{x\to 0+} \sqrt{x}\left(3+\sin\frac{1}{x}\right) = 0$

Find the following limits algebraically. Do not use the calculator.

4. (7pts) $\displaystyle\lim_{x\to\infty} \frac{x^2-4x+2}{2x+3} = \lim_{x\to\infty} \frac{x^2\left(1-\frac{4}{x}+\frac{2}{x^2}\right)}{x\left(2+\frac{3}{x}\right)} = \lim_{x\to\infty} x \cdot \frac{1-\frac{4}{x}+\frac{2}{x^2}}{2+\frac{3}{x}}$

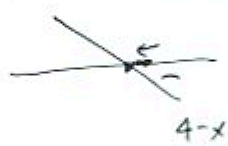
$$= \infty \cdot \frac{1-0+0}{2+0} = \infty \cdot \frac{1}{2} = \infty$$

5. (5pts) $\displaystyle\lim_{x\to 1} \frac{x-1}{x^2+5x-6} = \lim_{x\to 1} \frac{x-1}{(x-1)(x+6)} = \lim_{x\to 1} \frac{1}{x+6} = \frac{1}{7}$

6. (7pts) $\displaystyle\lim_{x\to 4} \frac{\sqrt{x}-2}{x-4} = \lim_{x\to 4} \frac{\sqrt{x}-2}{x-4} \cdot \frac{\sqrt{x}+2}{\sqrt{x}+2} = \lim_{x\to 4} \frac{\overset{\sqrt{x}^2-2^2}{\cancel{x-4}}}{\cancel{(x-4)}(\sqrt{x}+2)}$

$$= \lim_{x\to 4} \frac{1}{\sqrt{x}+2} = \frac{1}{4}$$

7. (6pts) $\displaystyle\lim_{x\to 4^+} \frac{x-1}{4-x} = \frac{4-1}{4-4} = \frac{3}{0^-} = -\infty$



8. (7pts) $\displaystyle\lim_{x\to 0} \frac{\sin(4x)\tan x}{x^2} = \lim_{x\to 0} \frac{\sin 4x}{x} \cdot \frac{\frac{\sin x}{\cos x}}{x} = \lim_{x\to 0} \frac{\sin 4x}{4x} \cdot 4 \cdot \frac{\sin x}{\cos x} \cdot \frac{1}{x}$

$$= \lim_{x\to 0} \frac{\sin 4x}{4x} \cdot 4 \cdot \frac{\sin x}{x} \cdot \frac{1}{\cos x} = 1 \cdot 4 \cdot 1 \cdot \frac{1}{1} = 4$$

9. (14pts) The equation $x^3 - 7 = \sqrt{x}$ is given.
a) Use the Intermediate Value Theorem to show it has a solution in the interval $(0, 3)$.
b) Use your calculator to find an interval of length at most 0.01 that contains a solution of the equation. Then use the Intermediate Value Theorem to justify why your interval contains the solution.

a) $x^3 - \sqrt{x} - 7 = 0$

$f(x) = x^3 - \sqrt{x} - 7$ is continuous

$f(0) = 0^3 - \sqrt{0} - 7 = -7$

$f(3) = 3^3 - \sqrt{3} - 7 = 20 - \sqrt{3} > 0$

Since $f(0) < 0 < f(3)$

by IVT there is a c in $(0,3)$

s.t. $f(c) = 0$

b) $f(2.03) = -0.059\ldots$

$f(2.04) = 0.061\ldots$

Since $f(2.03) < 0 < f(2.04)$, by IVT

there is a c in $(2.03, 2.04)$

s.t. $f(c) = 0$

10. (10pts) Consider the limit $\lim_{x \to 2} \dfrac{\sqrt{x} - \sqrt{2}}{x - 2}$. Use your calculator (don't forget parentheses) to estimate this limit with accuracy 3 decimal points. Write a table of values (no more than 5 per table) that will support your answer.

x	$\dfrac{\sqrt{x} - \sqrt{2}}{x - 2}$
2.1	0.34924
2.01	0.35311
2.001	0.35351
2.0001	0.35355
2.00001	0.35355

x	$\dfrac{\sqrt{x} - \sqrt{2}}{x - 2}$
1.9	0.35809
1.99	0.35400
1.999	0.3536
1.9999	0.35356
1.99999	0.353553

First three digits stabilize

It appears the limit is 0.353 to 3 decimal points

11. (12pts) Consider the function defined below.
a) Explain why the function is continuous on intervals $(0, 2)$ and $(2, \infty)$
b) For which c is the function continuous at point $x = 2$?

$$f(x) = \begin{cases} x^2 - cx, & \text{if } 0 < x < 2 \\ \frac{c}{x} + 5, & \text{if } x \geq 2. \end{cases}$$

a) f is a polynomial on $(0,2)$ and a rational function on $(2,\infty)$

b) For continuity need to have

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x)$$

$$\lim_{x\to 2^-} x^2 - cx = \lim_{x\to 2} \frac{c}{x} + 5$$

$$4 - 2c = \frac{c}{2} + 5$$

$$-\frac{5}{2}c = 1$$

$$c = -\frac{2}{5}$$

Bonus. (10pts) Find the limit algebraically. Do not use the calculator.

$$\lim_{h\to 0} \frac{(2+h)^4 - 16}{h} = \lim_{h\to 0} \frac{((2+h)^2)^2 - 4^2}{h} = \lim_{h\to 0} \frac{((2+h)^2 - 4)((2+h)^2 + 4)}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{(2+h-2)}(2+h+2)((2+h)^2+4)}{\cancel{h}} = (2+0+2)((2+0)^2+4)$$

$$= 4 \cdot 8 = 32$$

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[0]Total points: 100