Write interval notation and sketch on the number line.

1. (3pts) $\{x| -5 < x \le -1\}$

2. (3pts) $\{x| x < 3\}$

Solve the equations.

3. (3pts) $3x + 2 = 5x + 7$

4. (4pts) $3(a + 1) - 4 = 2a + 3(4a - 1)$

Simplify and write in standard form:

5. (4pts) $(2x + 1)^2 - 4x^2 + 3x =$

6. (4pts) $(5x + 3)x - (x + 2)(x + 7) =$

Simplify and write the answer so all exponents are positive:

7. (2pts) $(3a)^2 a^3 =$

8. (2pts) $\dfrac{x^7}{(4x)^2} =$

9. (3pts) $(a^{-3}b^2)^3 \, a^{10}b^{-7} =$

10. (5pts) $(u^4 v^{-3})^{-2}(2u^{-1}v^2)^4 =$

11. (7pts) $\dfrac{(3x^{-3}y^2)^4}{(6x^{-2}y^3)^2} =$

Factor the following.

12. (4pts) $x^2 - 2x - 15 =$ **13.** (4pts) $x^2 - 12x + 27 =$

Solve the equations.

**14.** (6pts) $x^2 - 4x = x + 24$ **15.** (6pts) $2x^2 + 2x + 28 = x^2 - 9x$

1. (13pts) Draw the triangle with vertices $A = (-2, -1)$, $B = (0, 3)$ and $C = (3, 1)$ in the coordinate plane.
a) Does it look like the triangle is a right triangle?
b) Compute the lengths of all sides of the triangle.
c) Use the Pythagorean theorem to determine algebraically whether ABC is a right triangle.

2. (8pts) Find the equation of the circle, if its center is $(-2, 1)$ and the point $(1, 4)$ is on the circle. Draw the circle.

3. (8pts) Use the graph of the function f at right to answer the following questions.

a) Find $f(-2)$ and $f(-5)$.

b) What is the domain of f?

c) What is the range of f?

d) What are the solutions
   of the equation $f(x) = 1$?
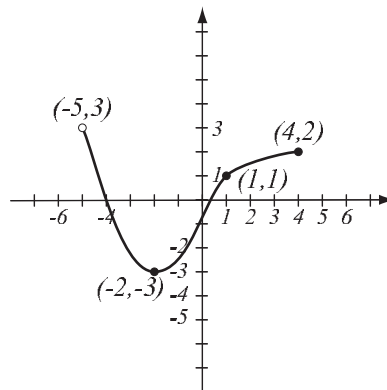
4. (12pts) The function
$f(x) = x^4 + 4x^3 + 9$ is given.

a) Use your calculator to accurately
its graph. Draw the graph here, and
indicate units on the axes.

b) Find all the $x-$ and $y-$intercepts
(accuracy: 6 decimal points).

c) State the domain and range.

5. (9pts) Find the domain of each function and write it using interval notation.

$$f(x) = \frac{2x + 3}{5x - 3}$$

$$g(x) = \sqrt{x} + \frac{1}{x - 9}$$

6. (10pts) Let $h(x) = \dfrac{x^2}{2x - 1}$. Find the following (simplify where appropriate).

$h(1) =$

$h(\frac{1}{2}) =$

$h(\sqrt{a}) =$

$h(x + 5) =$

1. (5pts) Find the equation of the line (in form $y = mx + b$) whose x-intercept is 4 and y-intercept is 3. Draw the graph of the line.

2. (10pts) Find the equation of the line (in form $y = mx + b$) passing through $(1, -5)$ that is perpendicular to the line $5x + 3y = 12$. Draw both lines.

3. (9pts) Draw the quadrilateral with vertices $A = (-1, -2)$, $B = (2, -3)$, $C = (4, 3)$ and $D = (1, 4)$.
a) Find the slopes of the sides of the quadrilateral.
b) Use slopes to check if the quadrilateral is a parallelogram (opposing sides parallel).
c) If the quadrilateral is a parallelogram, use slopes again to check if it is a rectangle.

4. (4pts) According to government data, the price (on average) of a pound of chicken breast was \$4.71 in August of 2022, and \$4.41 in April of 2023. What is the average rate of change of the price of a pound of chicken breast from August 2022 to April 2023? (Time is measured in months here.) What are the units for the average rate of change?

5. (12pts) The electric bill for a household was $125.28 in a month when it used 850 kWh (kilowatt-hours) of electricity. In another month, it used 1640 kWh and was billed $227.98.
a) Assuming that electricity cost $C(x)$ is a linear function of the amount of electricity x used (in kWh), write a formula for $C(x)$.
b) What is the cost if no electricity is used during a month? What is the meaning of this number?
c) What is the meaning of the slope in this example?

6. (20pts) An entomologist is trying to establish a connection between how quickly a certain species of a cricket chirps and the temperature. In the table, T is the temperature recorded in °F when a cricket was observed chirping x times in 15 seconds. Solve the problems below with accuracy 6 decimal points.
a) Draw the scatterplot of the data. Does the relationship look linear?
b) Use two points in the scatterplot to get an equation of a line that models the relationship between x and T. Draw the line on the graph.
c) Use your calculator to find the "line of best fit" for the data. Draw the line on the graph.
d) Find coefficient of correlation r. How strong is the linear relationship between x and T?
e) If we observe a cricket chirping 31 times in 15 seconds, what is the estimated temperature?

x	T
13	55
20	57
26	62
36	72
37	76
38	74

Solve the inequalities. Write your solution in interval notation.

1. (5pts) $-2 < 4 - 3x \leq 3$

2. (7pts) $2x + 1 < 7$ or $3x - 7 > 20$

3. (6pts) Find the domain of the function in interval notation: $f(x) = \dfrac{\sqrt{4x+7}}{x-3}$.

4. (14pts) Cab service Jiffy Jaunt charges $2.25 cents per mile for a ride. Their competitor Speedy Sis charges $7 for trips up to 2 miles, and then $2.05 per mile for miles past 2. Suppose the trips you are considering are longer than 2 miles. For which number of miles is Speedy Sis the better deal? Solve as an inequality.

5. (14pts) Hitchhiker Phil first catches a ride with a truck going 58 miles per hour. Luckily, shortly after he leaves the truck, he catches a ride in a car going 66 miles per hour, but rides in the car 30 minutes shorter than in the truck. After Phil leaves the car, he finds that he has traveled a total of 184 miles.

a) How much time did Phil spend in the truck and how much in the car ?

b) How far did he travel in the car?

6. (14pts) Rudy, Sidney and Jeffrey share the cost of their lawyers, for which the bill was $285,000. Sidney pays twice the amount Rudy and Jeffrey pay together, and Rudy pays $10,000 more than Jeffrey. How much does each of them contribute to the bill?

1. (10pts) Use your calculator to accurately sketch the graph of the function
$f(x) = \dfrac{x^3 + 4x^2 - 5x}{x^2 + 1}$. (When entering function into calculator, don't forget to put parentheses around numerator and denominator if the calculator doesn't have fractional notation.)
Draw the graph here, indicate units on the axes, and solve the problems below with accuracy 6 decimal points.
a) Find the local maxima and minima for this function.
b) State the intervals where the function is increasing and where it is decreasing.

2. (20pts) Let $f(x) = \dfrac{x + 2}{x - 2}$, $g(x) = \dfrac{3}{x}$. Find the following (simplify where possible):

$(f + g)(1) =$

$(fg)(5) =$

$\dfrac{f}{g}(x) =$

$(g \circ f)(3) =$

$(f \circ g)(x) =$

The domain of $(f + g)(x)$ in interval notation

3. (8pts) Consider the function $h(x) = \dfrac{4}{x^2 + x + 5}$ and find **two** different solutions to the following problem: find functions f and g so that $h(x) = f(g(x))$, where neither f nor g are the identity function.

4. (8pts) Sketch the graph of the piecewise-defined function:

$$f(x) = \begin{cases} -x + 3, & \text{if } x < 1 \\ 2x - 3 & \text{if } 1 \le x < 5. \end{cases}$$

5. (14pts) A devoper plans to build a block of four stores for a strip mall with a total area of 8000 square feet. Each store has an entrance door of width 6 feet. The developer wishes to minimize the construction cost, which is same as minimizing the total length of the walls.
a) Express the total length of the walls as a function of the length of one of the sides x. What is the domain of this function?
b) Graph the function in order to find the minimum. What are the dimensions of the block for which the total length of the walls is minimal? What is the minimal wall length?
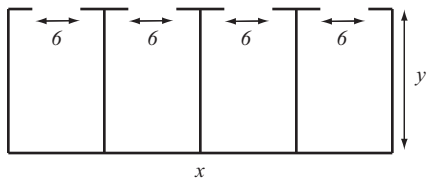
1. (21pts) For the following functions:
a) determine algebraically whether they are odd, even, or neither
b) use the calculator to draw their graphs here and verify your conclusions by stating symmetry.

$$f(x) = x^3 - 4x \qquad\qquad g(x) = |x| - 2x \qquad\qquad h(x) = x^4 - 5x^2 + 2$$

2. (16pts) Draw the graphs of $f(x) = 2\sqrt[3]{x-4}$ and $g(x) = -(3x)^2 + 1$ using transformations. Explain how you transform graphs of basic functions in order to get the graphs of f and g. Indicate at least two points on each graph.

3. (10pts) Write the equation for the function whose graph has the following characteristics:
a) shape of $y = |x|$, stretched vertically by factor 2,

b) shape of $y = \dfrac{1}{x}$, shifted left 3 units, then reflected about the x-axis,

c) shape of $y = \sqrt{x}$, reflected about the y-axis, then stretched horizontally by factor $\frac{1}{2}$, then shifted up 5.

4. (13pts) The graph of $f(x)$ is drawn below. On three separate graphs, sketch the graphs of the functions $f(x) + 1$, $2f(x-1)$ and $-f(3x)$ and label all the relevant points.
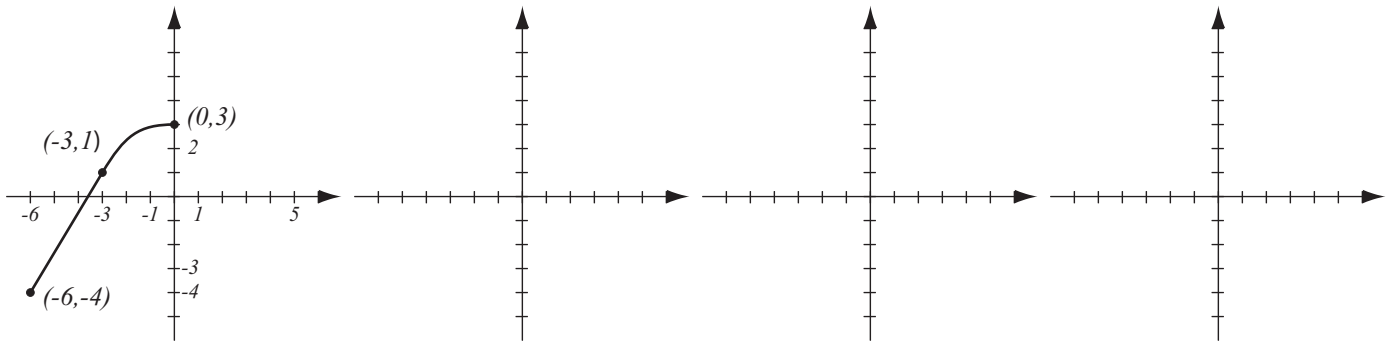
Use formulas to expand:

1. (4pts) $(6a - b)^2 =$

2. (5pts) $(3u + v^3)^2 =$

3. (4pts) $(4i - 3j)(4i + 3j) =$

Use the ac-method or another method to factor. Show how you got your answer.

4. (5pts) $4x^2 - 11x + 6 =$

5. (6pts) $18x^2 - 9x - 2 =$

6. (8pts) Compute expressions with fractions by hand.

$\dfrac{2}{5} \cdot \dfrac{25}{8} =$

$\dfrac{6}{7} \div \dfrac{9}{28} =$

$\dfrac{3}{2} - \dfrac{5}{6} =$

$\dfrac{7}{12} + \dfrac{5}{42} =$

Multiply or divide the rational expressions.

7. (7pts) $\dfrac{x-3}{2x+10} \cdot \dfrac{x^2-25}{3x^2-3x-18} =$

8. (7pts) $\dfrac{4x+10}{3x^2+4x-15} \div \dfrac{x^2-3x-18}{4x-24} =$

Add or subtract the rational expressions.

9. (6pts) $\dfrac{x+5}{x-2} - \dfrac{15x+12}{x^2+2x-8} =$

10. (8pts) $\dfrac{2x+1}{3x^2-5x-12} + \dfrac{x-4}{x^2+4x-21} =$

Simplify, so that the answer is in form $a + bi$.

1. (4pts) $(7 + 5i)^2 =$

2. (6pts) $\dfrac{-1 + 4i}{3 - 2i} =$

3. (4pts) Simplify and justify your answer.

$i^{111} =$

4. (8pts) The number of boxes of waffles in storage at a grocery store is given by the function $N(x) = -x^2 + 4x + 100$, where x is the number of days after October 28th.
a) On what dates did the store have 40 boxes for sale?
b) On what date did the number of boxes of waffles in storage reach its peak?

5. (8pts) Solve the equation: $x^4 + 13x^2 + 36 = 0$

6. (6pts) Solve by completing the square.

$x^2 - 10x + 4 = 0$

7. (12pts) The quadratic function $f(x) = -x^2 - 6x + 27$ is given. Do the following without using the calculator.

a) Find the x-intercepts of its graph, if any. Find the y-intercept.

b) Find the vertex of the graph.

c) Sketch the graph of the function.

8. (12pts) An artillery company is fighting in a rectangular theater of operations with dimensions 10 km by 14 km. Their guns have range 20 km, which is more than enough to hit any point in the rectangle from any other point. They would like to increase the theater by lengthening the 14 km side by a certain amount and the 10 km side by twice that amount, but so that any point in the new rectangle can be hit from any other point. By how much should they increase the 14 km side and the 10 km side? *Hint: in a rectangle, the largest distance between any two points is achieved by the diagonal. Set the problem up so that the diagonal of the expanded theater is 20 km.*

1. (4pts) Solve the equation.

$|3x - 5| = 4$

2. (12pts) Solve the inequalities. Draw your solution and write it in interval form.

$|x - 3| \geq 1$ $\qquad\qquad\qquad\qquad\qquad$ $|3x + 7| < 9$

Solve the equations:

3. (8pts) $\dfrac{2x}{x - 3} = \dfrac{x + 5}{x - 2} + \dfrac{18 - 4x}{x^2 - 5x + 6}$ $\qquad\qquad$ **4.** (8pts) $4 = x + \sqrt{52 - 3x}$

5. (14pts) An arrow is launched from height 4 meters upwards with initial velocity 55 meters per second. Its height in meters after t seconds is given by $s(t) = -5t^2 + 55t + 4$.
a) Sketch the graph of the height function.
b) When does the arrow reach its greatest height, and what is that height?
c) When is the arrow at height 140.8 meters?

6. (14pts) Truck mechanic Grayson wishes to build a repair shop with two side-by-side bays separated by a shorter wall (see picture). Grayson has enough money to build 1500 feet of walls, and he wants to build a shop with maximal area.
a) Express the total area of the shop as a function of one of the sides of the rectangle. What is the domain of this function?
b) Sketch the graph of the area function in order to find the maximum (no need for the graphing calculator — you should already know what the graph looks like). What are the dimensions of the shop that has the greatest total area? What is the greatest area possible?
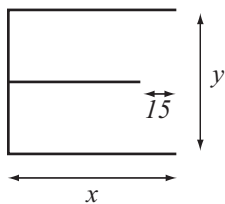
1. (6pts) The graph of a function f is given.
a) Is this function one-to-one? Justify.
b) If the function is one-to-one, find the graph
of f^{-1}, labeling the relevant points.
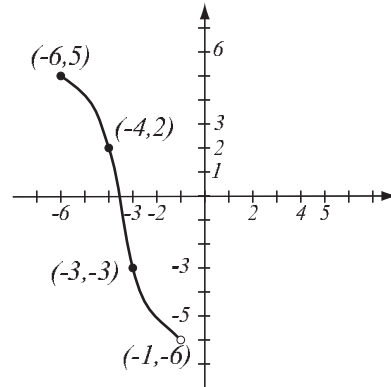
2. (12pts) Let $f(x) = \dfrac{3x - 1}{x + 4}$. Find the formula for f^{-1}. Find the ranges of f and f^{-1}.

3. (8pts) Evaluate without using the calculator. For each problem, write the question you
should ask yourself in order to find the logarithms.

$\log_6 36 =$ $\log_3 \frac{1}{81} =$ $\log_8 2 =$ $\log_{\sqrt{a}} a^3 =$

4. (4pts) Use the change-of-base formula
and your calculator to find $\log_4 5$ with ac-
curacy 6 decimal places. Show how you
obtained your number.

5. (12pts) Investigate the effect of increased frequency of compounding: for a deposit of $3,000 and annual interest rate of 4.74%, calculate the amount in the account after 1 year for the frequencies of compounding below.
a) Write the general formula for the amount, replacing the variables by numbers, if known.
b) Use the table feature on your calculator to quickly compute amounts after 1 year.
c) Does compounding more often make a big difference?

Frequency: every	n	Amount after 1 year
year		
quarter		
month		
day		
hour		
second		

6. (3pts) Find the domain of $f(x) = \ln(4x - 11)$.

7. (8pts) The cost per person of a field trip for x students is given by $C(x) = \dfrac{150 + 8x}{x}$, where C is in dollars.
a) Find the cost per person if 10 or 20 students go.
b) Find a formula for the inverse function and explain what it represents.
c) How many students need to go so that cost per person is $23? $13?

8. (7pts) Using transformations, draw the graph of $f(x) = -e^{x+4}$. Explain how you transform the graph of a basic function in order to get the graph of f. Show at least one point on the graph, and asymptotes to the graph, if any.

1. (5pts) If $\log_a 4 = 0.6667$ and $\log_a 9 = 1.0566$, calculate:

$\log_a \dfrac{9}{4} =$ $\log_a 144 =$

2. (11pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$\ln\left(e^4 x^5 y^{-3}\right) =$

$\log_5 \dfrac{\sqrt[4]{x^3} y^3}{25x^3} =$

3. (12pts) Write as a single logarithm. Simplify if possible.

$2\log_3(6x^2) - 4\log_3 y^3 - 3\log_3(2x^4) =$

$3\log(x+2) + 4\log(x-3) - 2\log(x^2 - x - 6) =$

4. (3pts) Simplify. $\qquad \log_9 9^{5-x} = \qquad\qquad\qquad 10^{\log(x-5)} =$

Solve the equations.

5. (5pts) $\left(\dfrac{1}{6}\right)^{3x-4} = 36^{2x}$ $\qquad\qquad\qquad\qquad$ **6.** (7pts) $8^{x+4} = 6^{3x}$

7. (5pts) A trucking company bought a truck for \$150K. The value of the truck each year is 92% of the value of the year before, so after t years its value in thousands is given by the function $V(t) = 150 \cdot 0.92^t$. When will the value of the truck be \$80K?

8. (12pts) The U.S population was 249 million in 1990 and 309 million in 2010. Assume the U.S. population grows exponentially.
a) Write the function describing the number $P(t)$ of people in the U.S. t years after 1990. Then find the exponential growth rate for this population.
b) Graph the function.
c) According to this model, when will the population reach 350 million?