College Algebra — Joysheet 11
MAT 140, Fall 2023 — D. Ivanšić

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Covers: 5.4-5.6 *Show all your work!*

1. (5pts) If $\log_a 4 = 0.6667$ and $\log_a 9 = 1.0566$, calculate:

$\log_a \frac{9}{4} =$ $\log_a 9 - \log_a 4$

$= 1.0566 - 0.6667$

$= 0.3899$

$\log_a 144 =$ $\log_a (4^2 \cdot 9)$

$= \log_a 4^2 + \log_a 9$

$= 2 \log_a 4 + \log_a 9$

$= 2 \cdot 0.6667 + 1.0566 = 2.39$

2. (11pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$\ln(e^4 x^5 y^{-3}) =$ $\ln e^4 + \ln x^5 + \ln y^{-3}$

$= 4 + 5\ln x - 3\ln y$

$\log_5 \frac{\sqrt[4]{x^3} y^3}{25x^3} =$ $\log_5 x^{\frac{3}{4}} + \log_5 y^3 - \log_5 25 - \log_5 x^3$

$= \frac{3}{4} \log_5 x + 3\log_5 y - 2 - 3\log_5 x$

$\frac{3}{4} - 3 = -\frac{9}{4}$

$= -\frac{9}{4} \log_5 x + 3\log_5 y - 2$

3. (12pts) Write as a single logarithm. Simplify if possible.

$2\log_3(6x^2) - 4\log_3 y^3 - 3\log_3(2x^4) =$ $\log_3 (6x^2)^2 - \log_3 (y^3)^4 - \log_3 (2x^4)^3$

$= \log_3 (36x^4) - \log_3 y^{12} - \log_3 (8x^{12})$

$= \log_3 \frac{\cancel{36}^{9} \cancel{x^4}}{y^{12} \cdot \cancel{8}_{2} x^{\cancel{12}\,8}} = \log_3 \frac{9}{2x^8 y^{12}}$

$3\log(x+2) + 4\log(x-3) - 2\log(x^2 - x - 6) =$

$= \log_3 (x+2)^3 + \log (x-3)^4 - \log (x^2 - x - 6)^2$

$= \log_3 \frac{(x+2)^3 (x-3)^4}{((x+2)(x-3))^2} = \log_3 \frac{(x+2)^{\cancel{3}\,1} (x-3)^{\cancel{4}\,2}}{\cancel{(x+2)^2}\,\cancel{(x-3)^2}} = \log_3 \left((x+2)(x-3)^2\right)$

4. (3pts) Simplify. $\log_9 9^{5-x} = 5-x$ $\quad 10^{\log(x-5)} = x-5$

Solve the equations.

5. (5pts) $\left(\frac{1}{6}\right)^{3x-4} = 36^{2x}$

$(6^{-1})^{3x-4} = (6^2)^{2x}$

$6^{-3x+4} = 6^{4x}$

so $-3x+4 = 4x$

$4 = 7x$

$x = \frac{4}{7}$

6. (7pts) $8^{x+4} = 6^{3x}$ $\quad | \ln$

$\ln 8^{x+4} = \ln 6^{3x}$

$(x+4)\ln 8 = 3x \ln 6$

$x\ln 8 + 4\ln 8 = 3x\ln 6$

$4\ln 8 = 3x\ln 6 - x\ln 8$

$4\ln 8 = x(3\ln 6 - \ln 8)$

$x = \frac{4\ln 8}{3\ln 6 - \ln 8} = 2.523719$

7. (5pts) A trucking company bought a truck for \$150K. The value of the truck each year is 92% of the value of the year before, so after t years its value in thousands is given by the function $V(t) = 150 \cdot 0.92^t$. When will the value of the truck be \$80K?

$150 \cdot 0.92^t = 80$

$0.92^t = \frac{80}{150}$ $\quad | \ln$

$\ln 0.92^t = \ln \frac{8}{15}$

$t \ln 0.92 = \ln \frac{8}{15}$

$t = \frac{\ln \frac{8}{15}}{\ln 0.92} = 7.538936$

In about 7.5 years

8. (12pts) The U.S population was 249 million in 1990 and 309 million in 2010. Assume the U.S. population grows exponentially.
a) Write the function describing the number $P(t)$ of people in the U.S. t years after 1990. Then find the exponential growth rate for this population.
b) Graph the function.
c) According to this model, when will the population reach 350 million?

a) $P(t) = 249e^{kt}$

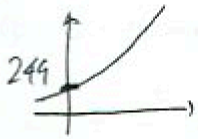
$309 = P(20) = 249e^{k \cdot 20}$

$\frac{309}{249} = e^{20k}$ $\quad | \ln$

$\ln \frac{309}{459} = 20k$

$k = \frac{\ln \frac{309}{459}}{20} = 0.0107944$

b)



c) $249e^{0.0107..t} = 350$

$e^{0.0107..t} = \frac{350}{249}$ $\quad | \ln$

$0.0107..t = \ln \frac{350}{249}$

$t = \frac{\ln \frac{350}{249}}{0.0107944} = 31.54225$

About 32 years After 1990, so in 2022.