College Algebra — Joysheet 10
MAT 140, Fall 2023 — D. Ivanšić

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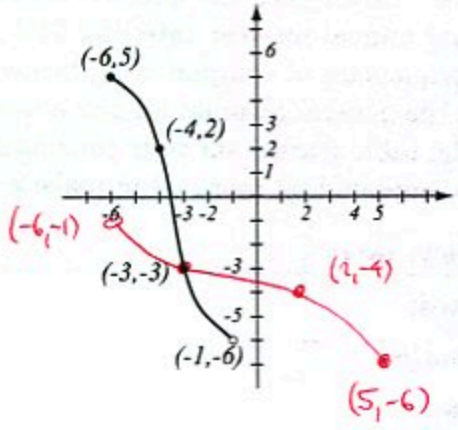
Covers: 5.1–5.3 *Show all your work!*

1. (6pts) The graph of a function f is given.
a) Is this function one-to-one? Justify.
b) If the function is one-to-one, find the graph of f^{-1}, labeling the relevant points.



a) Yes, it passes the horizontal line test.

2. (12pts) Let $f(x) = \dfrac{3x-1}{x+4}$. Find the formula for f^{-1}. Find the ranges of f and f^{-1}.

$y = \dfrac{3x-1}{x+4}$

$y(x+4) = 3x-1$

$yx+4y = 3x-1$

$yx-3x = -1-4y$

$x(y-3) = -1-4y$

$x = \dfrac{-1-4y}{y-3} = \dfrac{1+4y}{3-y} = f^{-1}(y)$

Range f = Domain f^{-1} can't have $3-y=0$, $y=3$

$(-\infty, 3) \cup (3, \infty)$

Range f^{-1} = Domain f can't have $x+4=0$, $x=-4$

$(-\infty, -4) \cup (-4, \infty)$

3. (8pts) Evaluate without using the calculator. For each problem, write the question you should ask yourself in order to find the logarithms.

$\log_6 36 = 2$ $\quad 6^? = 36$

$\log_3 \frac{1}{81} = -4$ $\quad 3^? = \frac{1}{81} = \frac{1}{3^4} = 3^{-4}$

$\log_8 2 = \frac{1}{3}$ $\quad 8^? = 2 = \sqrt[3]{8} = 8^{\frac{1}{3}}$

$\log_{\sqrt{a}} a^3 = 6$ $\quad (\sqrt{a})^? = a^3 = (a^{\frac{1}{2}})^6 = (\sqrt{a})^6$

4. (4pts) Use the change-of-base formula and your calculator to find $\log_4 5$ with accuracy 6 decimal places. Show how you obtained your number.

$\log_4 5 = \dfrac{\ln 5}{\ln 4} = 1.160964$

5. (12pts) Investigate the effect of increased frequency of compounding: for a deposit of \$3,000 and annual interest rate of 4.74%, calculate the amount in the account after 1 year for the frequencies of compounding below.
a) Write the general formula for the amount, replacing the variables by numbers, if known.
b) Use the table feature on your calculator to quickly compute amounts after 1 year.
c) Does compounding more often make a big difference?

Frequency: every	n	Amount after 1 year
year	1	3142.20
quarter	4	3144.75
month	12	3145.33
day	365	3145.61
hour	$365 \cdot 24 = 8760$	3145.62
second	$365 \cdot 24 \cdot 3600 = 31{,}536{,}000$	3145.62

b)

c) $A = 3000\left(1 + \frac{0.0474}{n}\right)^{n \cdot 1}$

$= 3000\left(1 + \frac{0.0474}{n}\right)^{n}$

c) Initially it makes a little difference, later, not very much.

6. (3pts) Find the domain of $f(x) = \ln(4x - 11)$.

must have $4x - 11 > 0$

$4x > 11$

$x > \frac{11}{4}$ $\quad \left(\frac{11}{4}, \infty\right)$

7. (8pts) The cost per person of a field trip for x students is given by $C(x) = \dfrac{150 + 8x}{x}$, where C is in dollars.
a) Find the cost per person if 10 or 20 students go.
b) Find a formula for the inverse function and explain what it represents.
c) How many students need to go so that cost per person is \$23? \$13?

a) $C(10) = \frac{150 + 80}{10} = 23$

$C(20) = \frac{150 + 160}{20} = 15.50$

b) $C = \frac{150 + 8x}{x}$

$Cx = 150 + 8x$

$Cx - 8x = 150$

$(C - 8)x = 150$

$\rightarrow x = \frac{150}{C - 8}$

Gives no. of participants needed so cost-per-person is C.

c) $x = \frac{150}{23 - 8} = \frac{150}{15} = 10$

$x = \frac{150}{13 - 8} = \frac{150}{5} = 30$

8. (7pts) Using transformations, draw the graph of $f(x) = -e^{x+4}$. Explain how you transform the graph of a basic function in order to get the graph of f. Show at least one point on the graph, and asymptotes to the graph, if any.