College Algebra — Joysheet 8 MAT 140, Fall 2023 — D. Ivanšić

Name: Saul Ocean

Covers: 3.1, 3.2, 3.3 Show all your work!

Simplify, so that the answer is in form $a + bi$.

1. (4pts) $(7+5i)^2 =$ $7^2 + 2\cdot 7\cdot 5i + (5i)^2 = 49 + 70i + 25i^2 = 24 + 70i$

-25 ; $= -1$

2. (6pts) $\dfrac{-1+4i}{3-2i} = \dfrac{-1+4i}{3-2i}\cdot\dfrac{3+2i}{3+2i} = \dfrac{-3-2i+12i+8i^2}{3^2-(2i)^2} = \dfrac{-3+10i-8}{9-4i^2} = \dfrac{-11+10i}{13}$

-8 ; $= +4$

3. (4pts) Simplify and justify your answer.

$i^{111} = i^{108}\cdot i^3 = 1\cdot i\cdot i\cdot i = -i$

$108 = 4\cdot 27$ so $i^{108} = (i^4)^{27} = 1^{27} = 1$; $= -1$

4. (8pts) The number of boxes of waffles in storage at a grocery store is given by the function $N(x) = -x^2 + 4x + 100$, where x is the number of days after October 28th.
a) On what dates did the store have 40 boxes for sale?
b) On what date did the number of boxes of waffles in storage reach its peak?

a) $-x^2 + 4x + 100 = 40$
$-x^2 + 4x + 60 = 0$
$x^2 - 4x - 60 = 0$
$(x-10)(x+6) =$

$x = 10, -6$
10 days after Oct 28 is Nov. 7th
6 days before Oct 28 is Oct 22nd

b) $N(x)$ ∩
$-\frac{b}{2a} = -\frac{4}{2\cdot(-1)} = 2$
2 days after Oct 28, on Oct 30th

5. (8pts) Solve the equation: $x^4 + 13x^2 + 36 = 0$

Let $u = x^2$ $\quad (x^2)^2 + 13x^2 + 36 = 0$
$u^2 + 13u + 36 = 0$
$(u+4)(u+9) = 0$
$u = -4, -9$

$x^2 = -4$ or $x^2 = -9$
$x = \pm\sqrt{-4}$ $\quad x = \pm\sqrt{-9}$
$= \pm 2i$ $\quad = \pm 3i$
(four solutions)

6. (6pts) Solve by completing the square.

$x^2 - 10x + 4 = 0$ $\quad 1 + 5^2$

$x^2 - 2\cdot 5\cdot x + 5^2 + 4 = 5^2$
$(x-5)^2 = 5^2 - 4$

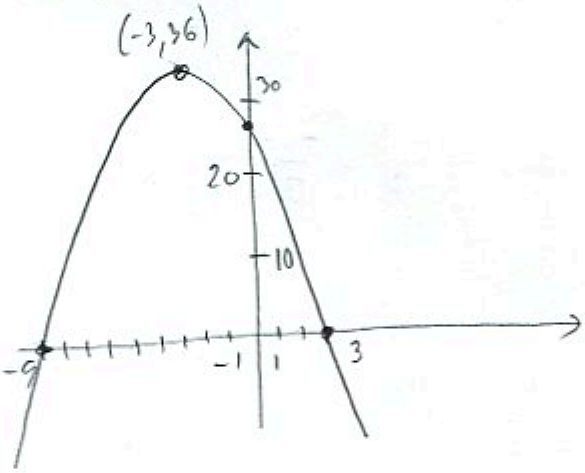
$(x-5)^2 = 21$
$x - 5 = \pm\sqrt{21}$
$x = 5 \pm \sqrt{21}$

7. (12pts) The quadratic function $f(x) = -x^2 - 6x + 27$ is given. Do the following without using the calculator.

a) Find the x-intercepts of its graph, if any. Find the y-intercept.

b) Find the vertex of the graph.

c) Sketch the graph of the function.



a) y-int: $f(0) = 27$

x-int: $-x^2 - 6x + 27 = 0$

$x^2 + 6x - 27 = 0$

$(x+9)(x-3) = 0$

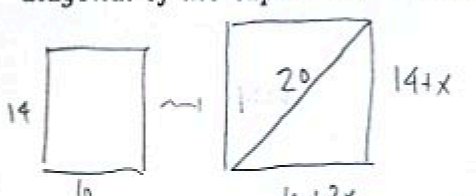
$x = -9, 3$

b) $h = -\frac{b}{2a} = -\frac{-6}{2(-1)} = -3$

$k = f(-3) = -(-3)^2 - 6(-3) + 27$

$= -9 + 18 + 27 = 36$

8. (12pts) An artillery company is fighting in a rectangular theater of operations with dimensions 10 km by 14 km. Their guns have range 20 km, which is more than enough to hit any point in the rectangle from any other point. They would like to increase the theater by lengthening the 14 km side by a certain amount and the 10 km side by twice that amount, but so that any point in the new rectangle can be hit from any other point. By how much should they increase the 14 km side and the 10 km side? *Hint: in a rectangle, the largest distance between any two points is achieved by the diagonal. Set the problem up so that the diagonal of the expanded theater is 20 km.*



$(10+2x)^2 + (14+x)^2 = 20^2$

$10^2 + 2 \cdot 10 \cdot 2x + (2x)^2 + 14^2 + 2 \cdot 14 \cdot x + x^2 = 400$

$100 + 40x + 4x^2 + 196 + 28x + x^2 = 400$

$5x^2 + 68x + 296 = 400 \quad | -400$

$5x^2 + 68x - 104 = 0$

$x = \frac{-68 \pm \sqrt{68^2 - 4 \cdot 5 \cdot (-104)}}{2 \cdot 5}$

$= \frac{-68 \pm \sqrt{6704}}{10} = \frac{-68 \pm 81.87}{10} = \begin{matrix} 1.387796 \\ -14.987796 \end{matrix}$

x, being length, can't be negative,

so $x = 1.387796$