1. (21pts) For the following functions:
a) determine algebraically whether they are odd, even, or neither
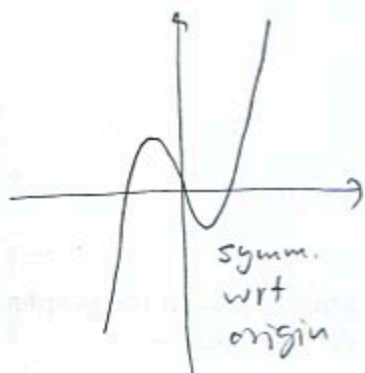b) use the calculator to draw their graphs here and verify your conclusions by stating symmetry.

$f(x) = x^3 - 4x$

$f(-x) = (-x)^3 - 4(-x)$

$\quad = -x^3 + 4x$
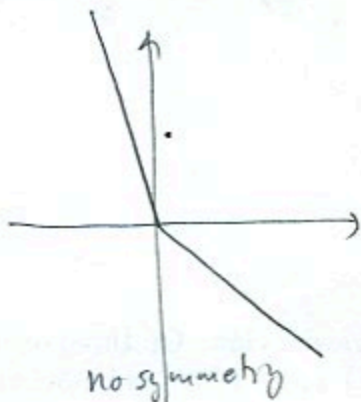
$\quad = -(x^3 - 4x)$

$\quad = -f(x)$ odd

$g(x) = |x| - 2x$

$g(-x) = |-x| - 2(-x)$

$\quad = |x| + 2x$

$\quad \neq g(x)$ neither
$\quad \neq -g(x)$

$h(x) = x^4 - 5x^2 + 2$

$h(-x) = (-x)^4 - 5(-x)^2 + 2$

$\quad = x^4 - 5x^2 + 2$
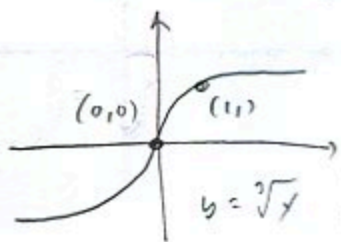
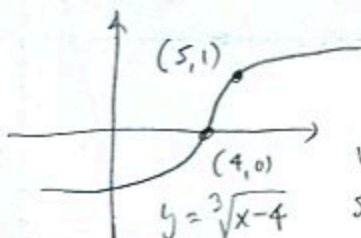even

symm. wrt origin

no symmetry

symm. wrt y-axis
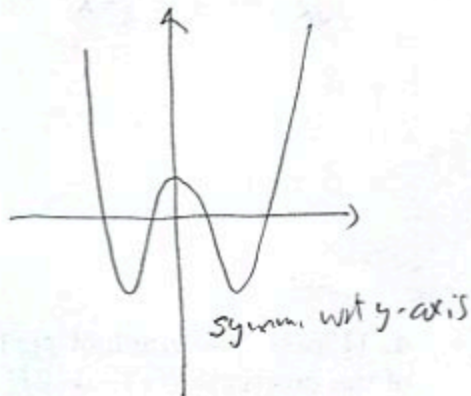
2. (16pts) Draw the graphs of $f(x) = 2\sqrt[3]{x-4}$ and $g(x) = -(3x)^2 + 1$ using transformations. Explain how you transform graphs of basic functions in order to get the graphs of f and g. Indicate at least two points on each graph.
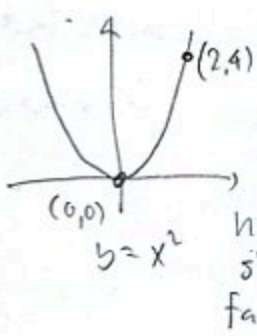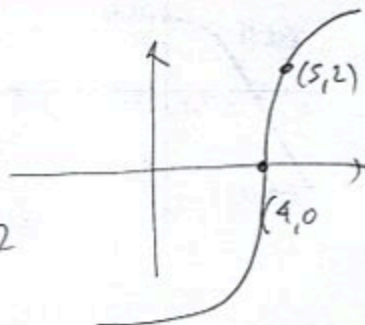

$(0,0)$ $(1,1)$ $y = \sqrt[3]{x}$ shift right 4

$(5,1)$ $(4,0)$ $y = \sqrt[3]{x-4}$ vertical stretch, Factor = 2

$(5,2)$ $(4,0)$

$(2,4)$ $(0,0)$ $y = x^2$ horiz. stretch, factor = $\frac{1}{3}$

$(\frac{2}{3}, 4)$ $(0,0)$ $y = (3x)^2$ reflect in x-axis

$(0,0)$ $(\frac{2}{3}, -4)$ $y = -(3x)^2$ shift up 1
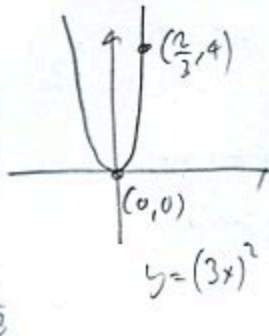
$(0,1)$ $(\frac{2}{3}, -3)$ $y = -(3x)^2 + 1$

3. (10pts) Write the equation for the function whose graph has the following characteristics:
a) shape of $y = |x|$, stretched vertically by factor 2,
b) shape of $y = \dfrac{1}{x}$, shifted left 3 units, then reflected about the x-axis,
c) shape of $y = \sqrt{x}$, reflected about the y-axis, then stretched horizontally by factor $\frac{1}{2}$, then shifted up 5.

$$|x| \rightsquigarrow 2|x|$$

$$\frac{1}{x} \rightsquigarrow \frac{1}{x+3} \rightsquigarrow -\frac{1}{x+3}$$

$$\sqrt{x} \rightsquigarrow \sqrt{-x} \rightsquigarrow \sqrt{-2x} \rightsquigarrow \sqrt{-2x} + 5$$

4. (13pts) The graph of $f(x)$ is drawn below. On three separate graphs, sketch the graphs of the functions $f(x) + 1$, $2f(x-1)$ and $-f(3x)$ and label all the relevant points.

(-3,1)   (0,3)   (-6,-4)

(0,9)   (-3,2)   (-6,-3)
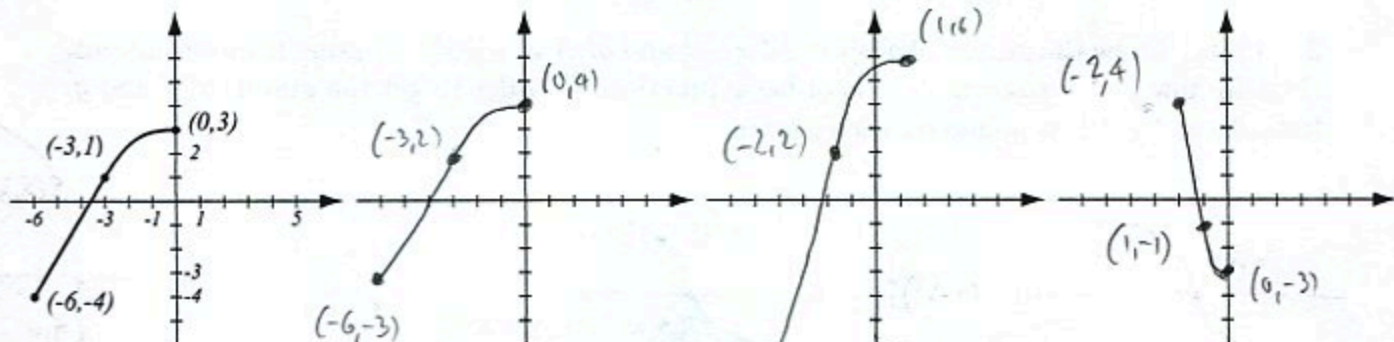
$f(x)+1$
shift up 1
$x \mapsto x$
$y \mapsto y+1$

(1,6)   (-2,2)   (-5,-8) $2f(x-1)$
shift right 1
vert. stretch, factor = 2
$x \mapsto x+1$
$y \mapsto 2y$

(-2,4)   (1,-1)   (0,-3) $-f(3x)$
horiz. stretch, factor = $\frac{1}{3}$
reflect in x-axis
$x \mapsto \frac{1}{3}x$
$y \mapsto -y$