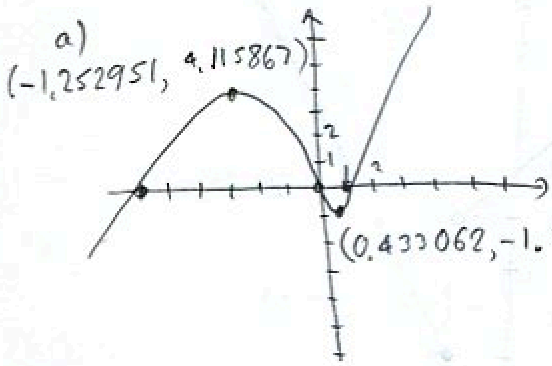
College Algebra — Joysheet 5 **MAT 140, Fall 2023 — D. Ivanšić**	**Name:** Saul Ocean *Covers: 2.1, 2.2, 2.3 Show all your work!*

1. (10pts) Use your calculator to accurately sketch the graph of the function $f(x) = \dfrac{x^3+4x^2-5x}{x^2+1}$. (When entering function into calculator, don't forget to put parentheses around numerator and denominator if the calculator doesn't have fractional notation.) Draw the graph here, indicate units on the axes, and solve the problems below with accuracy 6 decimal points.

a) Find the local maxima and minima for this function.

b) State the intervals where the function is increasing and where it is decreasing.



a) $4.115867 = f(-1.252951)$ is a local max

$-1.123262 = f(0.433062)$ is a local min.

b) Increasing on $(-\infty, -1.252951)$

and $(0.433062, \infty)$

Decreasing on $(-1.252951, 0.433062)$

2. (20pts) Let $f(x) = \dfrac{x+2}{x-2}$, $g(x) = \dfrac{3}{x}$. Find the following (simplify where possible):

$(f+g)(1) = f(1) + g(1)$

$= \dfrac{1+2}{1-2} + \dfrac{3}{1} = \dfrac{3}{-1} + 3 = 0$

$(fg)(5) = f(5)\cdot g(5)$

$= \dfrac{5+2}{5-2} \cdot \dfrac{3}{5} = \dfrac{7}{\cancel{3}} \cdot \dfrac{\cancel{3}}{5} = \dfrac{7}{5}$

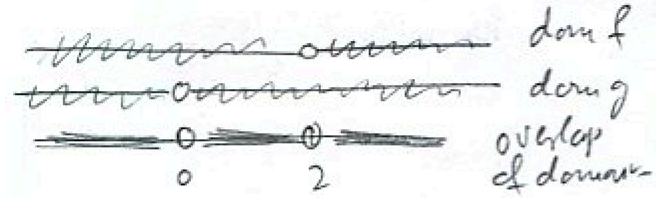
$\dfrac{f}{g}(x) = \dfrac{f(x)}{g(x)} = \dfrac{\frac{x+2}{x-2}}{\frac{3}{x}} = \dfrac{x+2}{x-2} \cdot \dfrac{x}{3} = \dfrac{x(x+2)}{3(x-2)}$

$(g \circ f)(3) = g(f(3)) = g\left(\dfrac{3+2}{3-2}\right)$

$= g\left(\dfrac{5}{1}\right) = g(5) = \dfrac{3}{5}$

$(f \circ g)(x) = f(g(x)) = f\left(\dfrac{3}{x}\right) = \dfrac{\frac{3}{x}+2}{\frac{3}{x}-2} \cdot \dfrac{x}{x} = \dfrac{3+2x}{3-2x}$

The domain of $(f+g)(x)$ in interval notation



Domain f: can't have $x-2=0$, $x=2$

Domain g: can't have $x=0$

Domain of $f+g$ is $(-\infty,0)\cup(0,2)\cup(2,\infty)$

3. (8pts) Consider the function $h(x) = \dfrac{4}{x^2+x+5}$ and find **two** different solutions to the following problem: find functions f and g so that $h(x) = f(g(x))$, where neither f nor g are the identity function.

$g(x) = x^2+x+5$ $\qquad$ $g(x) = x^2+x$

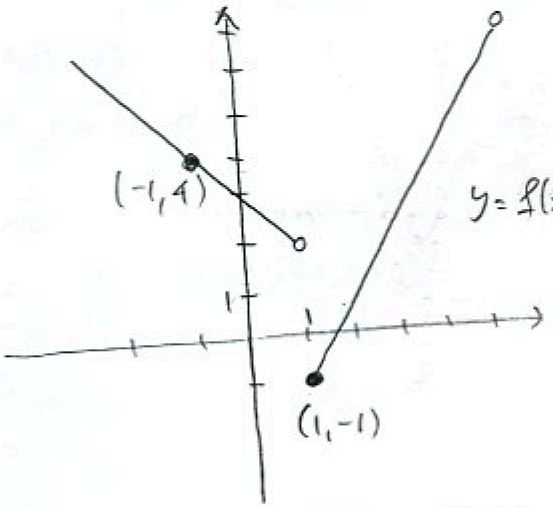
$f(x) = \dfrac{4}{x}$ $\qquad$ $f(x) = \dfrac{4}{x+5}$

4. (8pts) Sketch the graph of the piecewise-defined function:

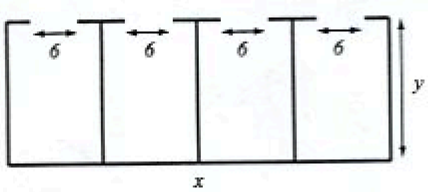
$$f(x) = \begin{cases} -x+3, & \text{if } x < 1 \\ 2x-3 & \text{if } 1 \le x < 5. \end{cases}$$

x	$-x+3$
1	2
-1	4

x	$2x-3$
1	-1
5	7



5. (14pts) A devoper plans to build a block of four stores for a strip mall with a total area of 8000 square feet. Each store has an entrance door of width 6 feet. The developer wishes to minimize the construction cost, which is same as minimizing the total length of the walls.
a) Express the total length of the walls as a function of the length of one of the sides x. What is the domain of this function?
b) Graph the function in order to find the minimum. What are the dimensions of the block for which the total length of the walls is minimal? What is the minimal wall length?



Total length of walls $= L = x + x - 4.6 + 5y$

$= 2x + 5y - 24$

$A = 8000$

$8000 = xy$

So $y = \dfrac{8000}{x}$

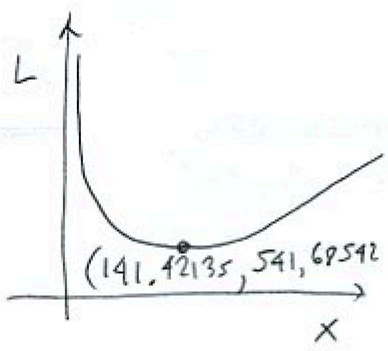
$= 2x + 5 \cdot \dfrac{8000}{x} - 24$

$L(x) = 2x + \dfrac{40000}{x} - 24$

Domain: Must have $x \geq 24$

No restriction on y.

Domain: $[24, \infty)$



dimensions are $\overset{x}{141.42135}$ by $\overset{y}{56.568545}$

$\dfrac{8000}{141.42135}$

Minimal wall length $= 541.68542$ ft