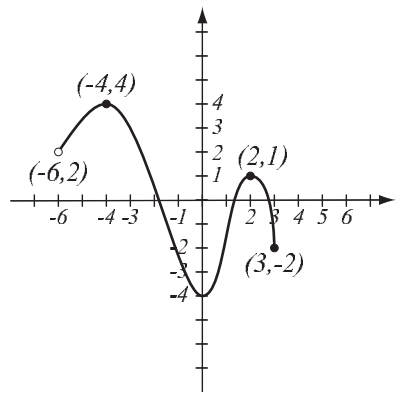
College Algebra — Exam 1
MAT 140C, Fall 2023 — D. Ivanšić

Name: _____

Show all your work!

1. (8pts) Use the graph of the function f at right to answer the following questions.



a) Find: $f(-4) =$ $\qquad$ $f(4) =$

b) What is the domain of f?

c) What is the range of f?

d) What are the solutions of the equation $f(x) = -3$?

2. (12pts) Use your calculator to accurately sketch the graph of $f(x) = x^3 - 8\sqrt{x+2}$.

a) Draw the graph on paper and indicate units on the axes.

b) Find all the $x-$ and $y-$intercepts (accuracy: 6 decimal points).

c) State the domain of the function in interval notation.

3. (5pts) Draw the line that passes through points $(-1, 7)$ and $(3, -1)$. Then write the equation of the line in form $y = mx + b$.

4. (10pts) Find the equation of the line (in form $y = mx + b$) that is perpendicular to the line $3x + 2y = 8$ and has x-intercept 3. Draw both lines.

5. (8pts) Draw the triangle with vertices $A = (-3, 0)$, $B = (-1, 3)$ and $C = (4, -1)$.
a) Find the lengths of the sides of the triangle.
b) Use the lengths of the sides to determine if this is a right triangle.

6. (10pts) Let $f(x) = x^2 + 3x - \sqrt{x} + 1$. Find the following (simplify where appropriate).

$f(1) =$

$f(-4) =$

$f(a^2) =$

$f(x - 2) =$

7. (6pts) Find the domain of the function below and write it using interval notation.

$$f(x) = \frac{2x + 1}{x^2 - 4x - 12}$$

8. (7pts) Solve and write the solution in interval notation.

$2x + 1 < 3$ or $3x - 5 > 10$

9. (8pts) A circle has center $(3, -2)$ and passes through the point $(2, 2)$.
a) Find the equation of the circle.
b) Draw the circle in the coordinate plane.

10. (12pts) Two cell phone companies charge the following monthly fees for mobile data:
— U-Mobile charges \$5 per gigabyte of data.
— BSS charges \$20 for the first 8 gigabytes, and \$7 per gigabyte for data over 8 gigabytes.
Assuming you always use more than 8 gigabytes of data, for which amounts of data is U-Mobile better? Solve as an inequality.

11. (14pts) A 159-mile road connects the Kentucky towns Princeton and Columbia. Pedro heads out from Princeton toward Columbia at 61mph. Jose heads out from Columbia toward Princeton 15 minutes later at 64mph. After a while, the friends meet on the road.
a) How long did each of them drive until they met?
b) How far from Princeton did they meet?

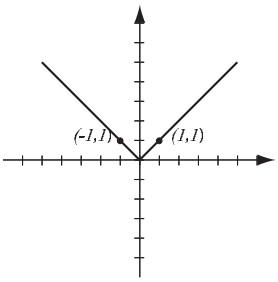
Bonus (10pts) Madison will invest some money into accounts bearing 3.5% and 4% simple interest. She plans to invest \$1,000 more into the account bearing 4% interest than the account bearing 3.5% interest. If she wishes to have at least \$193 in total interest after 1 year, what is the least she can invest in the account bearing 3.5% interest? Solve as an inequality.

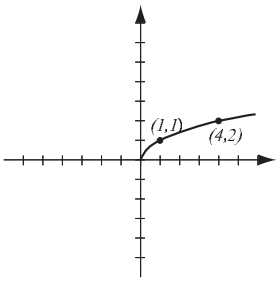
College Algebra — Exam 2 **MAT 140C, Fall 2023 — D. Ivanšić**

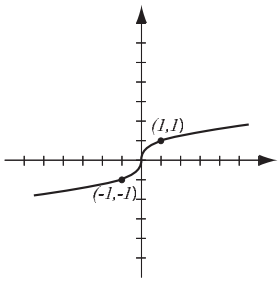
Name: _____

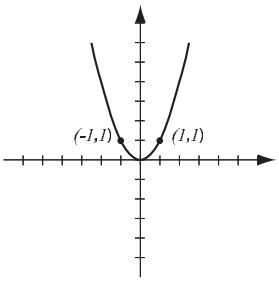
Show all your work!

1. (8pts) The following are graphs of basic functions. Write the equation of the graph under each one.









2. (20pts) Let $f(x) = \dfrac{x+3}{x-1}$, $g(x) = \sqrt{x+5}$.

Find the following (simplify where possible):

$(f-g)(2) =$

$(fg)(4) =$

$\dfrac{f}{g}(x) =$

$(g \circ f)(5) =$

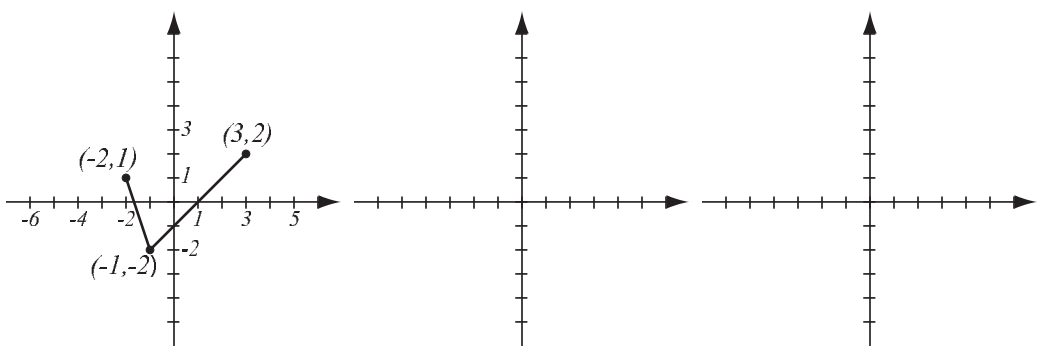
$(f \circ g)(x) =$

The domain of $f - g$ in interval notation

3. (6pts) Consider the function $h(x) = (4x^2 + 3)^3$ and find **two** different solutions to the following problem: find functions f and g so that $h(x) = f(g(x))$, where neither f nor g are the identity function.

4. (6pts) Write the equation for the function whose graph has the following characteristics:
a) shape of $y = \frac{1}{x}$, shifted left 3 units.
b) shape of $y = x^3$, stretched vertically by factor 3, then reflected over the y-axis.

5. (10pts) The graph of $f(x)$ is drawn below. Find the graphs of $f(-x) - 3$ and $-f\left(\frac{1}{2}x\right)$ and label all the relevant points.



6. (8pts) Sketch the graph of the piecewise-defined function:

$$f(x) = \begin{cases} 2x + 1, & \text{if } -3 \leq x < 3 \\ x - 4, & \text{if } x \geq 3 \end{cases}$$

7. (8pts) Find the values of the piecewise-defined function.

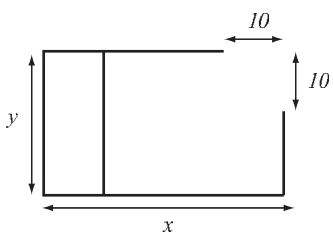
$$f(x) = \begin{cases} (x+1)^2, & \text{if } x < -2 \\ 4 - x, & \text{if } -2 \leq x < 4 \\ \sqrt{x+7}, & \text{if } 4 \leq x \leq 8 \end{cases}$$

$f(1) =$ $\qquad\qquad$ $f(-10) =$

$f(3+5) =$ $\qquad\qquad$ $f(42) =$

8. (20pts) Let $f(x) = x^3 - 3x^2 - 4x$ (answer with 6 decimal points accuracy).
a) Use your graphing calculator to accurately draw the graph of f (on paper!). Indicate units on the axes.
b) Determine algebraically whether the function is odd, even, or neither.
c) Verify your conclusion from b) by stating symmetry.
d) Find the local maxima and minima for this function. If there is symmetry, use it to reduce the work here.
e) State the intervals where the function is increasing and where it is decreasing.

9. (14pts) A drug store chain is building a store that is to have area 4000 square feet, a separated storage area (at left in picture), and doors at the corner, each of width 10 ft. To minimize cost, the total length of walls has to be as small as possible.
a) Express the total length of walls of the store as a function of the length of one of the sides x. What is the domain of this function?
b) Graph the function in order to find the minimum. What are the dimensions of the store that has the smallest total wall length? What is the smallest total wall length?



Bonus. (10pts) Using transformations and graphs of standard functions, sketch the graph of the piecewise-defined function:

$$f(x) = \begin{cases} x^3 + 2, & \text{if } x < -1 \\ x^2, & \text{if } -1 \leq x \leq 1 \\ 2 - \sqrt[3]{x}, & \text{if } x > 1 \end{cases}$$

College Algebra — Exam 3 **MAT 140C, Fall 2023 — D. Ivanšić**

Name: _____

Show all your work!

Simplify, so that the answer is in form $a + bi$.

1. (5pts) $4(2-3i) + 2i(-1+3i) =$

2. (5pts) $\dfrac{1+i}{-2+3i} =$

3. (4pts) Simplify and justify your answer.

$i^{86} =$

4. (6pts) Solve the equation by completing the square.

$x^2 + 10x = 24$

5. (6pts) Solve the inequality. Write the solution in interval form.

$|x-9| \geq 4$

6. (6pts) Let $P(x)$ be a polynomial of degree 3.
a) Draw a graph of P that has the maximal number of turning points.
b) Draw a graph of P that has the minimal number of intercepts.

7. (12pts) The quadratic function $f(x) = x^2 + 6x + 7$ is given. Do the following without using the calculator.
a) Find the x- and y-intercepts of its graph, if any.
b) Find the vertex of the graph.
c) Sketch the graph of the function.

Solve the equations:

8. (8pts) $\dfrac{x}{x-5} - \dfrac{2}{x-2} = \dfrac{18-6x}{x^2-7x+10}$

9. (8pts) $x = \sqrt{x+33} - 3$

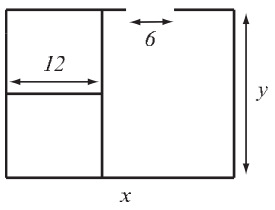
10. (14pts) The polynomial $f(x) = (x-5)^2(x+4)^2$ is given.
a) What is the end behavior of the polynomial?
b) List all the zeros and their multiplicities. Find the y-intercept.
c) Use the graphing calculator along with a) and b) to accurately sketch the graph of f (yes, on paper!).
d) Find all the turning points (i.e., local maxima and minima) with accuracy 6 decimal points.

11. (12pts) In a right triangle, one side is 2 cm shorter than the hypothenuse, and the other side is 3 cm shorter than the hypothenuse. What is the length of the hypothenuse?

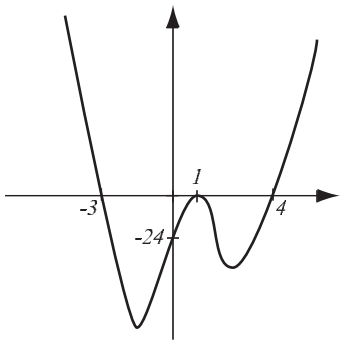
12. (14pts) Laura is designing a simple 3-room house with a 6-foot entrance door. To keep the home inexpensive, the budget allows for 120 feet of total wall length. Laura's goal is to maximize the total area of the house.

a) Express the total area of the house as a function of the length of one of the sides. What is the domain of this function?

b) Graph the function in order to find the maximum (no need for the graphing calculator — you should already know what the graph looks like). What are the dimensions of the house that has the biggest possible total area, and what is the biggest possible total area?



Bonus. (10pts) Write the formula of a polynomial whose graph is like the picture. *Hint: your formula has to have the prescribed x- and y-intercepts. Write it in factored form.*



College Algebra — Exam 4 **MAT 140C, Fall 2023 — D. Ivanšić**

Name: _____

Show all your work!

1. (8pts) Evaluate without using the calculator. For each problem, write the question you should ask yourself in order to find the logarithms.

$\log_4 64 =$ $\qquad$ $\log_5 \frac{1}{25} =$ $\qquad$ $\log_a \sqrt[4]{a^7} =$ $\qquad$ $\log_{a^2} a^8 =$

2. (4pts) Use the change-of-base formula and your calculator to find $\log_{19} 89$ with accuracy 6 decimal places. Show how you obtained your number.

3. (5pts) If $\log_a 4 = 1.262$ and $\log_a 7 = 1.771$, calculate the following values:

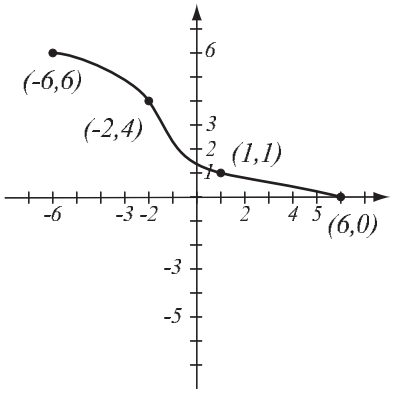
$\log_a 28 =$ $\qquad$ $\log_a \dfrac{4}{49} =$

4. (4pts) Simplify.

$\log 10^{x-1} =$ $\qquad$ $9^{\log_9 8080} =$

5. (8pts) If you deposit \$3,000 in an account bearing 4.5% interest, compounded monthly, how much is in the account after 2 years?

6. (6pts) The graph of a function f is given.
a) Is this function one-to-one? Justify.
b) If the function is one-to-one, find the graph of f^{-1}, labeling the relevant points, and showing any asymptotes.



7. (9pts) Let $f(x) = \dfrac{3x}{x-1}$.
a) Find the formula for f^{-1}.
b) Find the range of f^{-1}.

8. (6pts) Using transformations, draw the graph of $f(x) = e^{-x} - 2$. Explain how you transform the graph of a basic function in order to get the graph of f. Indicate at least one point on the graph and any asymptotes.

9. (12pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$\log\left(100x^5\sqrt[3]{y}\right) =$

$\log_2 \dfrac{x^3y^2}{8x^5} =$

10. (12pts) Write as a single logarithm. Simplify if possible.

$2\log(u^3v^{-2}) + 4\log(u^2v^3) =$

$4\log_2(x+4) - 2\log_2(x-4) - 2\log_2(x^2-16) =$

Solve the equations.

11. (6pts) $8^{x-3} = 2^{3-x}$

12. (8pts) $3^{2x+1} = 5^{3x}$

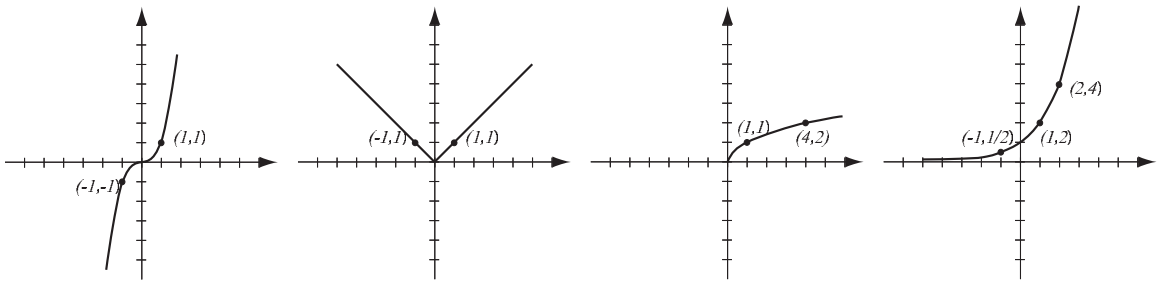
13. (12pts) According to census data, the population of McCracken County, KY, was 65,500 in 2000 and 67,900 in 2020. Assume that it has grown according to the formula $P(t) = P_0 e^{kt}$.
a) Find k and write the function that describes the population at time t years since 2000. Graph it on paper.
b) Find the predicted population in the year 2040.

Bonus (10pts) Solve the equation.

$\log_4(x+1) + \log_4(x+7) = 2$

College Algebra — Final Exam MAT 140C, Fall 2023 — D. Ivanšić	**Name:** _____ *Show all your work!*

1. (8pts) The following are graphs of basic functions. Write the equation of the graph under each one.



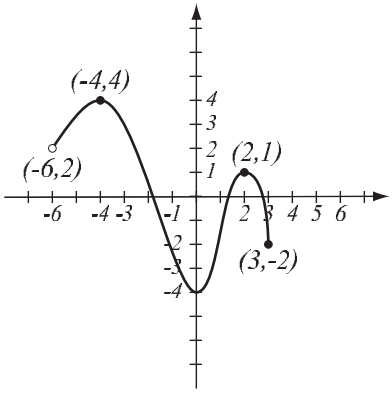
2. (8pts) Use the graph of the function f at right to answer the following questions.

a) Find: $f(-4) =$ $\qquad f(4) =$

b) What is the domain of f?

c) What is the range of f?

d) What are the solutions of the equation $f(x) = 1$?



3. (5pts) Find the equation of the line that passes through points $(3, -2)$ and $(1, 6)$.

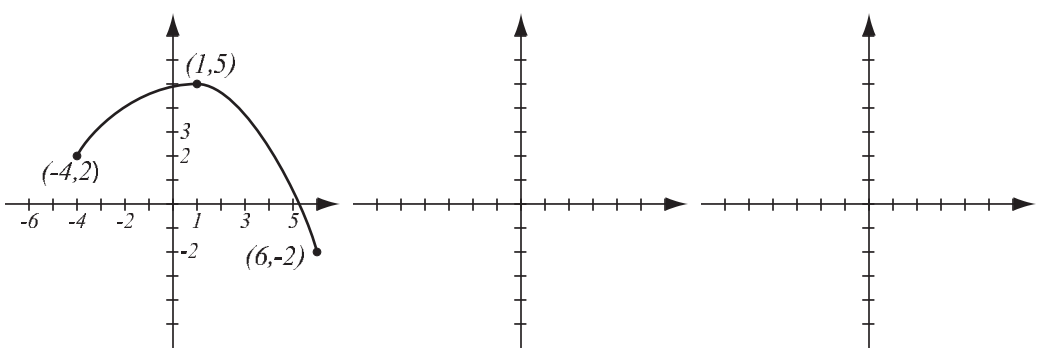
4. (7pts) Draw the triangle with vertices $A = (-2, 1)$, $B = (6, 0)$ and $C = (0, 4)$. Use either the Pythagorean theorem and lengths of sides or use slopes to determine if this is a right triangle.

5. (3pts) Find the domain of the function $f(x) = \dfrac{1}{3x+6}$ and write it in interval notation.

6. (6pts) Solve and write the solution in interval notation.

$|x-5| > 3$

7. (10pts) The graph of $f(x)$ is drawn below. Find the graphs of $-f(x+1)$ and $f(2x)-3$ and label all the relevant points.



8. (6pts) Let $f(x) = \dfrac{3x}{x-1}$. Find the formula for f^{-1}.

9. (12pts) The quadratic function $f(x) = x^2 - 2x - 15$ is given. Do the following without using the calculator.
a) Find the x- and y-intercepts of its graph, if any.
b) Find the vertex of the graph.
c) Sketch the graph of the function.

10. (5pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$\log_8 \left(64x^5 \sqrt[4]{y}\right) =$

11. (6pts) Write as a single logarithm. Simplify if possible.

$3\ln(u^{-2}v^3) - 2\ln(u^5v^3) =$

12. (6pts) Let $f(x) = x^2 + 2x$, $g(x) = x - 3$. Find the following (simplify where possible):

$(f + g)(x) =$ $\qquad\qquad$ $(f \circ g)(x) =$

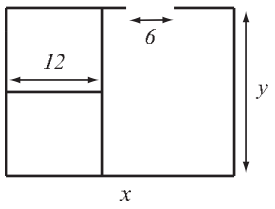
13. (20pts) The polynomial $P(x) = x^2(x - 4)(x + 4)$ is given (answer with 6 decimals accuracy).
a) What is the end behavior of the polynomial?
b) Find all the zeros and their multiplicities. Find the y-intercept.
c) Determine algebraically whether the function is odd, even, or neither. (Multiply out the factors if you need to.)
d) Use the graphing calculator along with a) and b) to sketch the graph of P (yes, on paper!).
e) Verify your conclusion from c) by stating symmetry.
f) Find all the turning points (i.e., local maxima and minima).

14. (8pts) Solve the equation.

$$\frac{x-3}{x+2} + \frac{3x}{x-8} = \frac{3x^2+48}{x^2-6x-16}$$

15. (14pts) A 166-mile road connects the Kentucky towns Princeton and Columbia. Pedro heads out from Princeton toward Columbia at 66mph. Jose heads out from Columbia toward Princeton a quarter of an hour later at 72mph. After a while, the friends meet on the road.
a) How long did each of them drive until they met?
b) How far from Princeton did they meet?

16. (14pts) Laura is designing a simple 3-room house with a 6-foot entrance door. To keep the home inexpensive, the budget allows for 150 feet of total wall length. Laura's goal is to maximize the total area of the house.
a) Express the total area of the house as a function of the length of one of the sides. What is the domain of this function?
b) Graph the function in order to find the maximum (no need for the graphing calculator — you should already know what the graph looks like). What are the dimensions of the house that has the biggest possible total area, and what is the biggest possible total area?



17. (12pts) According to census data, the population of McCracken County, KY, was 65,500 in 2000 and 67,900 in 2020. Assume that it has grown according to the formula $P(t) = P_0 e^{kt}$.
a) Find k and write the function that describes the population at time t years since 2000. Graph it on paper.
b) Find the predicted population in the year 2030.

Bonus (10pts) Solve the equation.

$\log_4(x+1) + \log_4(x+7) = 2$