## College Algebra - Final Exam MAT 140C, Spring 2023 - D. Ivanšić

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1. (8pts) The following are graphs of basic functions. Write the equation of the graph under each one.

2. (8pts) Use the graph of the function $f$ at right to answer the following questions.
a) Find: $f(-4)=\quad f(2)=$
b) What is the domain of $f$ ?
c) What is the range of $f$ ?
d) What are the solutions of the equation $f(x)=-2$ ?

3. (10pts)
a) Find the equation of the line that passes through points $(1,-1)$ and $(4,5)$.
b) Find the equation of the line (in form $y=m x+b$ ) that is parallel to the line in a) and and passes through the point $(-2,0)$.
c) Draw both lines.
4. (3pts) Find the domain of the function $f(x)=\sqrt{2 x-5}$ and write it in interval notation.
5. ( 6 pts ) Solve and write the solution in interval notation.
$|x+8| \leq 2$
6. (10pts) The graph of $f(x)$ is drawn below. Find the graphs of $f(-x)+2$ and $2 f(x+3)$ and label all the relevant points.

7. (6pts) The graph of a function $f$ is given.
a) Is this function one-to-one? Justify.
b) If the function is one-to-one, find the graph of $f^{-1}$, labeling the relevant points.

8. (12pts) The quadratic function $f(x)=x^{2}+4 x+7$ is given. Do the following without using the calculator.
a) Find the $x$ - and $y$-intercepts of its graph, if any.
b) Find the vertex of the graph.
c) Sketch the graph of the function.
9. (5pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.
$\log _{5}\left(125 x^{7} \sqrt[3]{y}\right)=$
10. (6pts) Write as a single logarithm. Simplify if possible.
$2 \log \left(x^{4} y^{2}\right)-4 \log \left(x^{3} y\right)=$
11. (8pts) Let $f(x)=\frac{x}{x^{2}-5}, g(x)=\sqrt{x-3}$. Find the following (simplify where possible):
$\frac{f}{g}(x)=$

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(f \circ g)(x)=
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12. (20pts) The polynomial $P(x)=x^{3}-4 x$ is given (answer with 6 decimals accuracy).
a) What is the end behavior of the polynomial?
b) Factor the polynomial to find all the zeros and their multiplicities. Find the $y$-intercept.
c) Determine algebraically whether the function is odd, even, or neither.
d) Use the graphing calculator along with a ) and b ) to sketch the graph of $P$ (yes, on paper!).
e) Verify your conclusion from c) by stating symmetry.
f) Find all the turning points (i.e., local maxima and minima).
13. (8pts) Solve the equation.
$\frac{2 x}{x+4}+\frac{10 x-8}{x^{2}+2 x-8}=\frac{x}{x-2}$
14. (14pts) A truck enters a highway, and a car does the same a half an hour later and drives in the same direction as the truck. Because the car drives 17 miles per hour faster than the truck, it catches up with the truck an hour and a half after the car entered.
a) What are the speeds of the truck and the car?
b) How far from the entrance of the highway did they meet?
15. (14pts) Spiffy Dude is building a quick-oil-change shop with four bays. It has budgeted enough money to build 240 feet of walls, and its goal is to maximize the total area of the shop.
a) Express the total area of the shop as a function of the length of one of the sides. What is the domain of this function?
b) Graph the function in order to find the maximum (no need for the graphing calculator you should already know what the graph looks like). What are the dimensions of the shop that has the biggest possible total area, and what is the biggest possible total area?

16. (12pts) According to census data, the population of Lexington, KY, was 296,000 in 2010 and 323,000 in 2020. Assume that it has grown according to the formula $P(t)=P_{0} e^{k t}$. a) Find $k$ and write the function that describes the population at time $t$ years since 2010. Graph it on paper.
b) Find the predicted population in the year 2027.

Bonus (10pts) The natural gas bill for a household was $\$ 54.76$ in a month when it used 18 hcf of gas. In another month, it used 29 hcf and was billed $\$ 74.67$. (Hcf is a unit for quantity of natural gas used.)
a) Assuming that gas cost $C(x)$ is a linear function of the amount of gas $x$ used (in hcf), write a formula for $C(x)$.
b) What is the meaning of the slope in this example?

