## College Algebra - Exam 4 MAT 140C, Spring 2023 - D. Ivanšić

Show all your work!

1. (8pts) Evaluate without using the calculator. For each problem, write the question you should ask yourself in order to find the logarithms.
$\log _{3} 27=$
$\log _{2} \frac{1}{8}=$
$\log _{a} \sqrt[3]{a^{4}}=$
$\log _{\sqrt{a}} a^{6}=$
2. (4pts) Use the change-of-base formula and your calculator to find $\log _{4} 54$ with accuracy 6 decimal places. Show how you obtained your number.
3. (5pts) If $\log _{a} 3=1.098$ and $\log _{a} 5=1.609$, calculate the following values:
$\log _{a} 15=$
$\log _{a} \frac{25}{3}=$
4. (4pts) Simplify.
$\ln e^{|x|-1}=$ $7^{\log _{7} 1331}=$
5. ( 8 pts ) If you deposit $\$ 2500$ in an account bearing $4.1 \%$ interest, compounded quarterly, how much is in the account after 4 years?
6. (6pts) The graph of a function $f$ is given.
a) Is this function one-to-one? Justify.
b) If the function is one-to-one, find the graph of $f^{-1}$, labeling the relevant points, and showing any asymptotes.

7. (9pts) Let $f(x)=\frac{x}{x+3}$.
a) Find the formula for $f^{-1}$.
b) Find the range of $f$.
8. (6pts) Using transformations, draw the graph of $f(x)=-\ln (x+4)$. Explain how you transform the graph of a basic function in order to get the graph of $f$. Indicate at least one point on the graph and any asymptotes.
9. (12pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.
$\ln \left(e^{2} x^{3} \sqrt{y}\right)=$
$\log _{3} \frac{9 x^{2} y^{4}}{x y^{6}}=$
10. (12pts) Write as a single logarithm. Simplify if possible.
$3 \log \left(w^{3} z^{2}\right)+2 \log \left(w^{2} z^{-4}\right)=$
$4 \log _{2}(x+5)+3 \log _{2}(x-1)-2 \log _{2}\left(x^{2}+4 x-5\right)=$

Solve the equations.
11. $(6 \mathrm{pts}) ~ 16^{2 x+1}=4^{x+3}$
12. $(8 \mathrm{pts}) 4^{x}=7^{1-2 x}$
13. (12pts) According to census data, the population of Lexington, KY, was 296,000 in 2010 and 323,000 in 2020. Assume that it has grown according to the formula $P(t)=P_{0} e^{k t}$. a) Find $k$ and write the function that describes the population at time $t$ years since 2010 . Graph it on paper.
b) Find the predicted population in the year 2030.

Bonus (10pts) Solve the equation.
$\log _{2}(x-3)+\log _{2}(x+1)=5$

