College Algebra — Exam 4
MAT 140C, Fall 2023 — D. Ivanšić

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Show all your work!

1. (8pts) Evaluate without using the calculator. For each problem, write the question you should ask yourself in order to find the logarithms.

$\log_4 64 = 3$ $\quad 4^? = 64$

$\log_5 \frac{1}{25} = -2$ $\quad 5^? = \frac{1}{25} = \frac{1}{5^2} = 5^{-2}$

$\log_a \sqrt[4]{a^7} = \frac{7}{4}$ $\quad a^? = a^{\frac{7}{4}}$

$\log_{a^2} a^8 = 4$ $\quad (a^2)^? = a^8$

2. (4pts) Use the change-of-base formula and your calculator to find $\log_{19} 89$ with accuracy 6 decimal places. Show how you obtained your number.

$\log_{19} 89 = \frac{\ln 89}{\ln 19} = 1.524445$

3. (5pts) If $\log_a 4 = 1.262$ and $\log_a 7 = 1.771$, calculate the following values:

$\log_a 28 = \log_a(4 \cdot 7)$
$= \log_a 4 + \log_a 7$
$= 1.262 + 1.771$
$= 3.033$

$\log_a \frac{4}{49} = \log_a\left(\frac{4}{7^2}\right)$
$= \log_a 4 - 2\log_a 7$
$= 1.262 - 2 \cdot 1.771$
$= -2.28$

4. (4pts) Simplify.

$\log 10^{x-1} = x - 1$

$9^{\log_9 8080} = 8080$

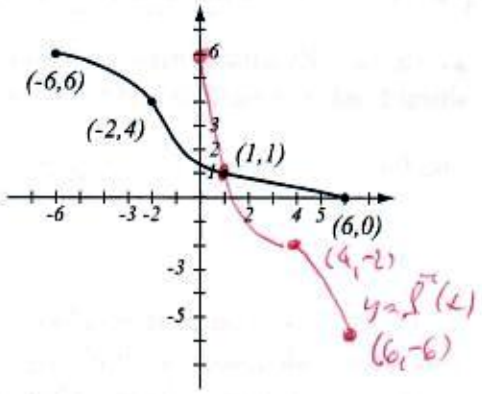
5. (8pts) If you deposit \$3,000 in an account bearing 4.5% interest, compounded monthly, how much is in the account after 2 years?

$A = P\left(1 + \frac{r}{n}\right)^{nt} = 3000\left(1 + \frac{0.045}{12}\right)^{12 \cdot 2}$

$= 3000 \cdot 1.09399\ldots$

$= 3281.99$

6. (6pts) The graph of a function f is given.
a) Is this function one-to-one? Justify.
b) If the function is one-to-one, find the graph of f^{-1}, labeling the relevant points, and showing any asymptotes.



a) Yes, it passes the horizontal line test

7. (9pts) Let $f(x) = \dfrac{3x}{x-1}$.
a) Find the formula for f^{-1}.
b) Find the range of f^{-1}.

Range of f^{-1} = Domain of f $= (-\infty, 1) \cup (1, \infty)$

$f(x) = \dfrac{3x}{x-1}$ Can't have $x-1=0$, $x=1$

$y = \dfrac{3x}{x-1}$

$y(x-1) = 3x$

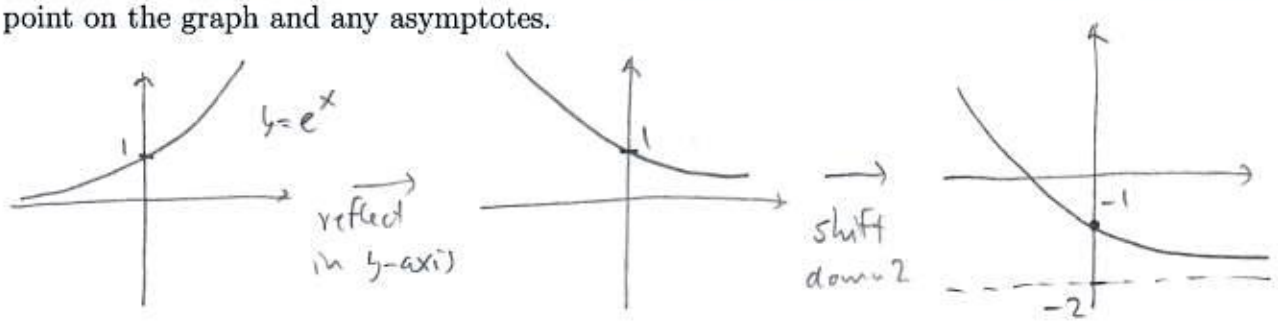
$yx - y = 3x$

$yx - 3x = y$

$x(y-3) = y$

$x = \dfrac{y}{y-3} = f^{-1}(y)$

8. (6pts) Using transformations, draw the graph of $f(x) = e^{-x} - 2$. Explain how you transform the graph of a basic function in order to get the graph of f. Indicate at least one point on the graph and any asymptotes.



9. (12pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$\log\left(100x^5\sqrt[3]{y}\right) = \log 100 + \log x^5 + \log y^{\frac{1}{3}}$

$= 2 + 5\log x + \frac{1}{3}\log y$

$\log_2 \frac{x^3y^2}{8x^5} = \log_2 x^3 + \log_2 y^2 - \log_2 8 - \log_2 x^5$

$= 3\log_2 x + 2\log_2 y - 3 - 5\log_2 x$

$= 2\log_2 y - 2\log_2 x - 3$

10. (12pts) Write as a single logarithm. Simplify if possible.

$2\log(u^3v^{-2}) + 4\log(u^2v^3) = \log(u^3v^{-2})^2 + \log(u^2v^3)^4$

$= \log\left((u^3v^{-2})^2 \cdot (u^2v^3)^4\right) = \log\left(u^6v^{-4}u^8v^{12}\right) = \log\left(u^{14}v^8\right)$

$4\log_2(x+4) - 2\log_2(x-4) - 2\log_2(x^2-16) = \log_2(x+4)^4 - \log_2(x-4)^2 - \log_2(x^2-16)^2$

$= \log_2 \frac{(x+4)^4}{(x-4)^2(x^2-16)^2} = \log_2 \frac{(x+4)^{4-2}}{(x-4)^2(x+4)^2(x-4)^2} = \log_2 \frac{(x+4)^2}{(x-4)^4}$

$(x^2-16) = (x+4)(x-4)$

Solve the equations.

11. (6pts) $8^{x-3} = 2^{3-x}$

$(2^3)^{x-3} = 2^{3-x}$

$2^{3x-9} = 2^{3-x}$

$3x-9 = 3-x \quad |+x+9$

$4x = 12$

$x = 3$

12. (8pts) $3^{2x+1} = 5^{3x} \quad |\ln$

$\ln 3^{2x+1} = \ln 5^{3x}$

$(2x+1)\ln 3 = 3x \ln 5$

$2x\ln 3 + \ln 3 = 3x\ln 5$

$\ln 3 = 3x\ln 5 - 2x\ln 3$

$\ln 3 = x(3\ln 5 - 2\ln 3)$

$x = \frac{\ln 3}{3\ln 5 - 2\ln 3} = 0.41755$

13. (12pts) According to census data, the population of McCracken County, KY, was 65,500 in 2000 and 67,900 in 2020. Assume that it has grown according to the formula $P(t) = P_0e^{kt}$.
a) Find k and write the function that describes the population at time t years since 2000. Graph it on paper.
b) Find the predicted population in the year 2040.

a) $P(t) = P_0e^{kt}$ $\quad t=0$

$P(t) = 655e^{kt}$ (in hundreds) for 2000

$679 = P(20) = 655e^{k\cdot 20}$

$\frac{679}{655} = e^{20k}$ $\quad |\ln$

$\ln\frac{679}{655} = 20k$

$k = \frac{\ln\frac{679}{655}}{20} = 0,00179929$

$P(t) = 655e^{0.00179929t}$

b) $P(40) = 655e^{0,0017..\ 40}$

$= 703.879389$

About 70,388 people in 2040.

Bonus (10pts) Solve the equation.

$\log_4(x+1) + \log_4(x+7) = 2$

$\log_4((x+1)(x+7)) = 2$

$4^{\log_4((x+1)(x+7))} = 4^2$

$(x+1)(x+7) = 16$

$x^2+8x+7 = 16$

$x^2+8x-9 = 0$

$(x+9)(x-1) = 0$

$x = -9,\ 1$

$-9+1<0,\ -9+7<0$ so -9 is not a sol.

$1+1>0,\ 1+7>0$ so 1 is a sol.

Only $\boxed{x=1}$ is a solution