College Algebra — Exam 3 **MAT 140C, Fall 2023 — D. Ivanšić**	**Name:** Saul Ocean *Show all your work!*

Simplify, so that the answer is in form $a + bi$.

1. (5pts) $4(2-3i) + 2i(-1+3i) = 8 - 12i - 2i + 6i^2$

$= 8 - 14i - 6 = 2 - 14i$

2. (5pts) $\dfrac{1+i}{-2+3i} = \dfrac{1+i}{-2+3i} \cdot \dfrac{-2-3i}{-2-3i} = \dfrac{-2-2i-3i-3i^2}{(-2)^2-(3i)^2} = \dfrac{-2-5i+3}{4-9i^2} = \dfrac{1-5i}{13}$

$= \dfrac{1}{13} - \dfrac{5}{13}i$

3. (4pts) Simplify and justify your answer.

$i^{86} = i^{84} \cdot i^2 = (\underbrace{i^4}_{=1})^{21} \cdot (-1) = -1$

$=1$

4. (6pts) Solve the equation by completing the square.

$x^2 + 10x = 24 \quad | + 5^2$

$x^2 + 2 \cdot x \cdot 5 + 5^2 = 24 + 5^2$

$(x+5)^2 = 49$

$x + 5 = \pm 7$

$x = -5 \pm 7 = -12, 2$

5. (6pts) Solve the inequality. Write the solution in interval form.

$|x - 9| \geq 4$

distance from x to $9 \geq 4$

$-4 \quad +4$

5 9 13

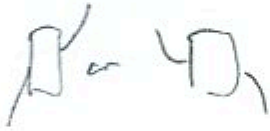
$(-\infty, 5] \cup [13, \infty)$

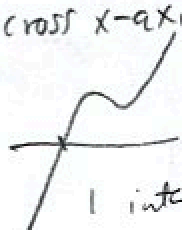
6. (6pts) Let $P(x)$ be a polynomial of degree 3.
a) Draw a graph of P that has the maximal number of turning points.
b) Draw a graph of P that has the minimal number of intercepts.

a)

max turning pts = 2

b) Has to be like - must cross x-axis at least once





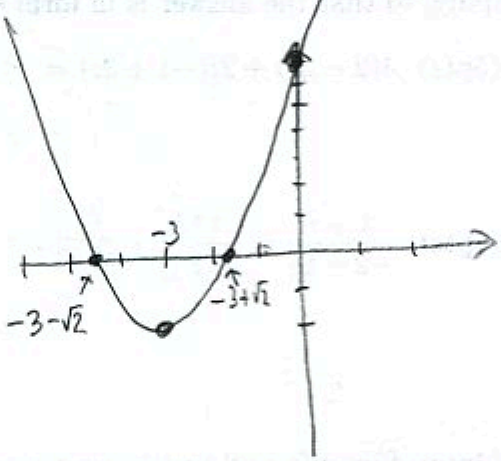
1 intercept

7. (12pts) The quadratic function $f(x) = x^2 + 6x + 7$ is given. Do the following without using the calculator.

a) Find the x- and y-intercepts of its graph, if any.

b) Find the vertex of the graph.

c) Sketch the graph of the function.



a) y-int: $f(0)=7$

x-int: $x^2+6x+7=0$

$$x = \frac{-6 \pm \sqrt{6^2-4\cdot1\cdot7}}{2\cdot1}$$

$$= \frac{-6 \pm \sqrt{8}}{2} = \frac{-6 \pm 2\sqrt{2}}{2}$$

$$= \frac{2(-3\pm\sqrt{2})}{2} = -3+\sqrt{2},\ -3-\sqrt{2}$$

$$\approx -1.6 \quad \approx -4.4$$

vertex: $-\frac{b}{2a} = -\frac{6}{2} = -3$

$$f(-3) = (-3)^2 + 6\cdot(-3) + 7$$

$$= 9 - 18 + 7 = -2$$

Solve the equations:

8. (8pts) $\dfrac{x}{x-5} - \dfrac{2}{x-2} = \dfrac{18-6x}{x^2-7x+10}$ $\Big|\cdot(x-5)(x-2)$

$(x-5)(x-2)$

$$\frac{x}{\cancel{x-5}}\cancel{(x-5)}(x-2) - \frac{2}{\cancel{x-2}}(x-5)\cancel{(x-2)} = \frac{18-6x}{\cancel{(x-5)(x-2)}}\cancel{(x-5)(x-2)}$$

$x(x-2) - 2(x-5) = 18-6x$

$x^2-2x-2x+10 = 18-6x \quad | +6x-18$

$x^2-4x+6x+10-18=0$

$x^2+2x-8=0$

$(x+4)(x-2)=0$

$x=-4, \cancel{2}$ — gives 0 in denom

$x=-4$

9. (8pts) $x = \sqrt{x+33} - 3$

$x+3 = \sqrt{x+33} \quad |^2$

$(x+3)^2 = x+33$

$x^2 + 2\cdot x\cdot 3 + 3^2 = x+33$

$x^2+6x+9 = x+33 \quad | -x-33$

$x^2+5x-24=0$

$(x+8)(x-3)=0$

$x=-8, 3$

Check: $-8 \stackrel{?}{=} \sqrt{-8+33}-3$

$-8 \stackrel{?}{=} \sqrt{25}-3$ no

$3 = \sqrt{3+33}-3$

$3 = \sqrt{36}-3$ yes

Only $x=3$ is the solution

10. (14pts) The polynomial $f(x) = (x-5)^2(x+4)^2$ is given.
a) What is the end behavior of the polynomial?
b) List all the zeros and their multiplicities. Find the y-intercept.
c) Use the graphing calculator along with a) and b) to accurately sketch the graph of f (yes, on paper!).
d) Find all the turning points (i.e., local maxima and minima) with accuracy 6 decimal points.

a) Like $(x)^2(x)^2 = x^4$ ∪

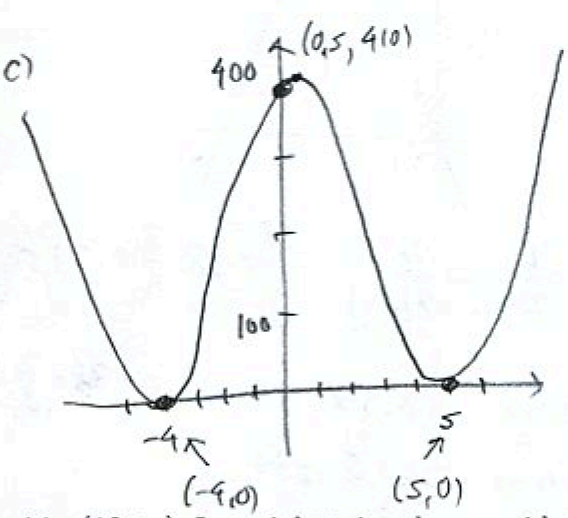
b)

zeros	5	-4
mult	2	2

touch

y-int: $f(0) = (-5)^2 \cdot 4^2 = 400$

c)

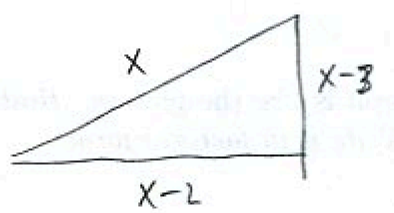


d) local max is $410 = f(0.5)$

local minimae $0 = f(-4)$

$0 = f(5)$

11. (12pts) In a right triangle, one side is 2 cm shorter than the hypothenuse, and the other side is 3 cm shorter than the hypothenuse. What is the length of the hypothenuse?



$(x-2)^2 + (x-3)^2 = x^2$

$x^2 - 2\cdot x\cdot 2 + 2^2 + x^2 - 2\cdot x\cdot 3 + 3^2 = x^2$

$x^2 - 4x + 4 + x^2 - 6x + 9 = x^2 \quad | -x^2$

$x^2 - 10x + 13 = 0$ $\quad 100-52 \quad 16\cdot 3$

$x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4\cdot 1\cdot 13}}{2\cdot 1} = \frac{-10 \pm \sqrt{48}}{2} = \frac{-10 \pm 4\sqrt{3}}{2}$

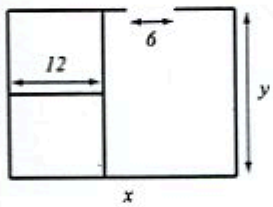
$= \frac{2(-5 \pm 2\sqrt{3})}{2} = -5 \pm 2\sqrt{3}$

$x = -5 + 2\sqrt{3} = -1.53 < 0$

$-5 - 2\sqrt{3} < 0$

Both solutions are negative, so no solution.

12. (14pts) Laura is designing a simple 3-room house with a 6-foot entrance door. To keep the home inexpensive, the budget allows for 120 feet of total wall length. Laura's goal is to maximize the total area of the house.
a) Express the total area of the house as a function of the length of one of the sides. What is the domain of this function?
b) Graph the function in order to find the maximum (no need for the graphing calculator — you should already know what the graph looks like). What are the dimensions of the house that has the biggest possible total area, and what is the biggest possible total area?



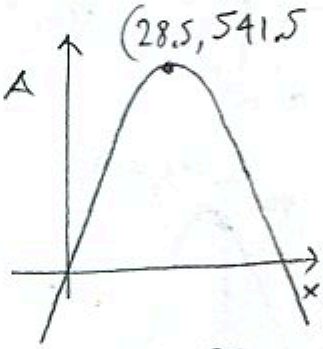
$A = xy = x\left(38 - \frac{2}{3}x\right) = -\frac{2}{3}x^2 + 38x$

$x + 12 + x - 6 + 3y = 120$

$2x + 3y + 6 = 120$

$3y = 114 - 2x$

$y = \frac{114 - 2x}{3} = 38 - \frac{2}{3}x$



Domain:

Must have: $x \geq 12 + 6$

$x \geq 18$

$y \geq 0$

$38 - \frac{2}{3}x \geq 0$

$\frac{2}{3}x \leq 38 \quad | \cdot \frac{3}{2}$

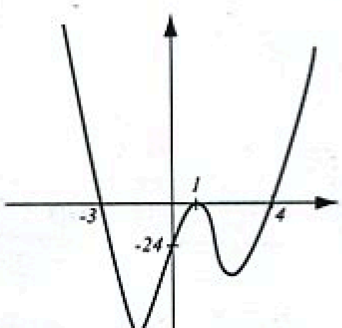
$x \leq 57$

$-\frac{b}{2a} = -\frac{38}{2\left(-\frac{2}{3}\right)} = \frac{38}{\frac{4}{3}} = 38 \cdot \frac{3}{4} = 28.5$

Dimensions: 28.5×19 $\leftarrow 38 - \frac{2}{3} \cdot 28.5$

Max area: 541.5 sq.ft

Bonus. (10pts) Write the formula of a polynomial whose graph is like the picture. *Hint: your formula has to have the prescribed x- and y-intercepts. Write it in factored form.*



$f(x) = C(x+3)(x-1)^2(x-4)$

$-24 = f(0) = C \cdot 3 \cdot (-1)^2 \cdot (-4)$

$-24 = C \cdot (-12)$

$C = \frac{-24}{-12} = 2$

$f(x) = 2(x+3)(x-1)^2(x-4)$