College Algebra — Exam 2
MAT 140C, Fall 2023 — D. Ivanšić

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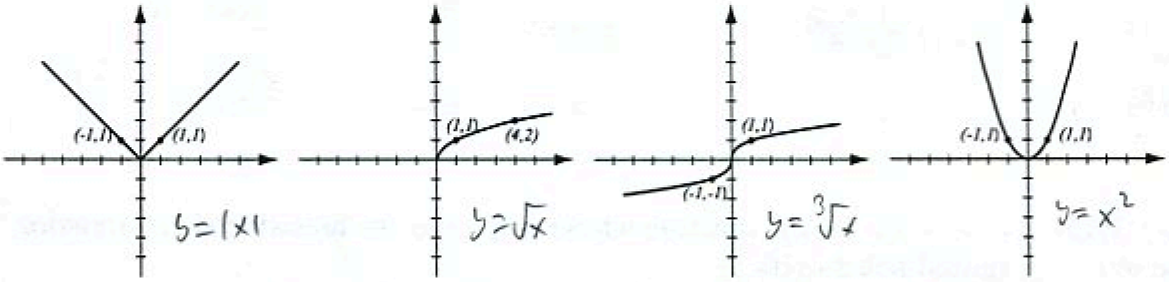
Show all your work!

1. (8pts) The following are graphs of basic functions. Write the equation of the graph under each one.



$y = |x|$ $\qquad$ $y = \sqrt{x}$ $\qquad$ $y = \sqrt[3]{x}$ $\qquad$ $y = x^2$

2. (20pts) Let $f(x) = \dfrac{x+3}{x-1}$, $g(x) = \sqrt{x+5}$.

Find the following (simplify where possible):

$(f-g)(2) = f(2) - g(2)$

$= \frac{2+3}{2-1} - \sqrt{2+5} = 5 - \sqrt{7}$

$(fg)(4) = f(4) \cdot g(4) = \frac{4+3}{4-1} \cdot \sqrt{4+5}$

$= \frac{7}{3} \cdot \sqrt{9} = \frac{7}{3} \cdot 3 = 7$

$\frac{f}{g}(x) = \frac{f(x)}{g(x)} = \frac{\frac{x+3}{x-1}}{\sqrt{x+5}} = \frac{x+3}{x-1} \cdot \frac{1}{\sqrt{x+5}}$

$= \frac{x+3}{(x-1)\sqrt{x+5}}$

$(g \circ f)(5) = g(f(5)) = g\left(\frac{5+3}{5-1}\right) = g\left(\frac{8}{4}\right)$

$= g(2) = \sqrt{2+5} = \sqrt{7}$

$(f \circ g)(x) = f(g(x)) = f(\sqrt{x+5}) = \frac{\sqrt{x+5}+3}{\sqrt{x+5}-1}$

The domain of $f-g$ in interval notation

Domain f: can't have $x - 1 = 0$

$x = 1$

Domain g: must have $x + 5 \geq 0$

$x \geq -5$

dom f

dom g

dom(f−g)

−5 $\qquad$ 1

= overlap

Domain of f−g $[-5, 1) \cup (1, \infty)$

3. (6pts) Consider the function $h(x) = (4x^2+3)^3$ and find **two** different solutions to the following problem: find functions f and g so that $h(x) = f(g(x))$, where neither f nor g are the identity function.

some possibilities

$g(x) = 4x^2+3$, $f(x) = x^3$

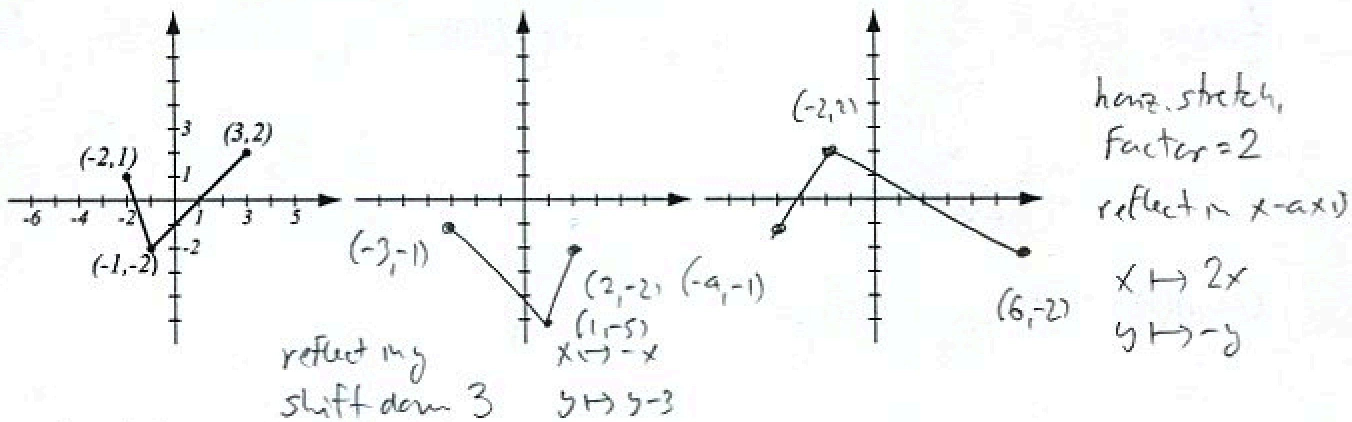
$g(x) = 4x^2$, $f(x) = (x+3)^3$

$g(x) = x^2$, $f(x) = (4x+3)^3$

4. (6pts) Write the equation for the function whose graph has the following characteristics:
a) shape of $y = \frac{1}{x}$, shifted left 3 units.
b) shape of $y = x^3$, stretched vertically by factor 3, then reflected over the y-axis.

a) $\frac{1}{x} \rightsquigarrow \frac{1}{x+3}$ b) $x^3 \rightsquigarrow 3x^3 \rightsquigarrow 3(-x)^3$

5. (10pts) The graph of $f(x)$ is drawn below. Find the graphs of $f(-x)-3$ and $-f\left(\frac{1}{2}x\right)$ and label all the relevant points.

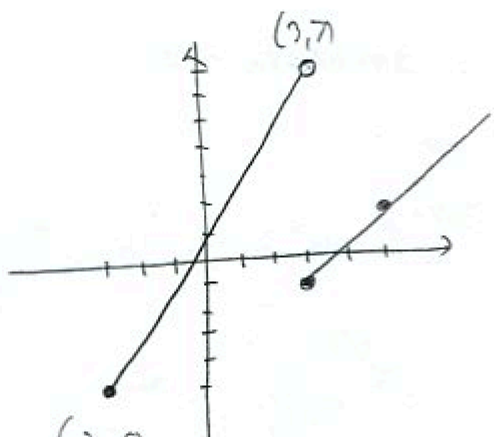


6. (8pts) Sketch the graph of the piecewise-defined function:

$$f(x) = \begin{cases} 2x+1, & \text{if } -3 \le x < 3 \\ x-4, & \text{if } x \geqslant 3 \end{cases}$$

x	$2x+1$
-3	-5
3	7

x	$x-4$
3	-1
5	1



7. (8pts) Find the values of the piecewise-defined function.

$$f(x) = \begin{cases} (x+1)^2, & \text{if } x < -2 \\ 4 - x, & \text{if } -2 \le x < 4 \\ \sqrt{x+7}, & \text{if } 4 \le x \le 8 \end{cases}$$

$f(1) =$ $4-1=3$

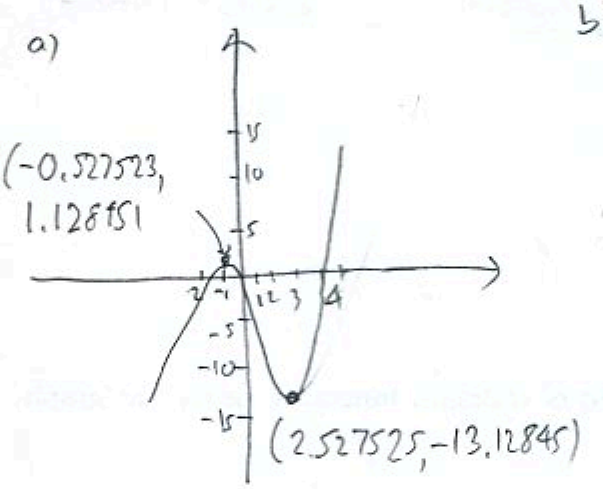
$f(-10) =$ $(-10+1)^2=(-9)^2=81$

$f(3+5) =$ $f(8)=\sqrt{8+7}=\sqrt{15}$

$f(42) =$ not defined

8. (20pts) Let $f(x) = x^3 - 3x^2 - 4x$ (answer with 6 decimal points accuracy).
a) Use your graphing calculator to accurately draw the graph of f (on paper!). Indicate units on the axes.
b) Determine algebraically whether the function is odd, even, or neither.
c) Verify your conclusion from b) by stating symmetry.
d) Find the local maxima and minima for this function. If there is symmetry, use it to reduce the work here.
e) State the intervals where the function is increasing and where it is decreasing.

a)



b) $f(-x) = (-x)^3 - 3(-x)^2 - 4(-x)$
$= -x^3 - 3x^2 + 4x \neq f(x)$
$\neq -f(x)$

neither

c) No symmetry

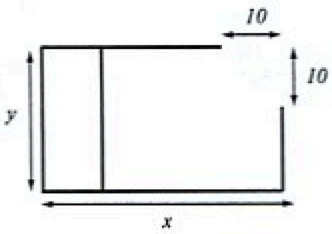
d) Local min is $-13.12845 = f(2.527525)$

Local max is $1.128451 = f(-0.527523)$

e) increasing on $(-\infty, -0.527523)$ and $(2.527525, \infty)$

decreasing on $(-0.527523, 2.527525)$

9. (14pts) A drug store chain is building a store that is to have area 4000 square feet, a separated storage area (at left in picture), and doors at the corner, each of width 10 ft. To minimize cost, the total length of walls has to be as small as possible.
a) Express the total length of walls of the store as a function of the length of one of the sides x. What is the domain of this function?
b) Graph the function in order to find the minimum. What are the dimensions of the store that has the smallest total wall length? What is the smallest total wall length?



$\ell = x + x - 10 + y + y + y - 10 = 2x + 3y - 10$

$A = 4000 = xy \quad , \quad y = \frac{4000}{x}$

$\ell = 2x + 3 \cdot \frac{4000}{x} - 10 = 2x + \frac{12000}{x} - 10$

Domain:

Must have

$x \geq 10$

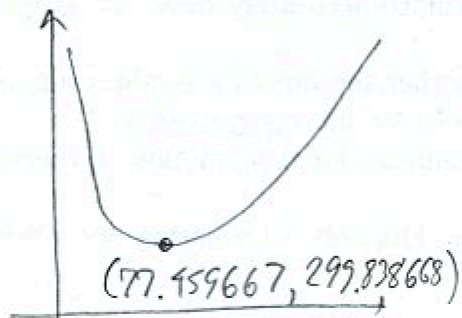
$y \geq 10$

$\frac{4000}{x} \geq 10$

$4000 \geq 10x$

$400 \geq x$

Dom $[10, 400]$



$\frac{4000}{77.459...}$

Dimensions:

77.459667 ft x 51.639778 ft

Minimal total wall length

299.838668 ft

Bonus. (10pts) Using transformations and graphs of standard functions, sketch the graph of the piecewise-defined function:

$$f(x) = \begin{cases} x^3 + 2, & \text{if } x < -1 \\ x^2, & \text{if } -1 \leq x \leq 1 \\ 2 - \sqrt[3]{x}, & \text{if } x > 1 \end{cases}$$