1. (8pts) Use the graph of the function f at right to answer the following questions.
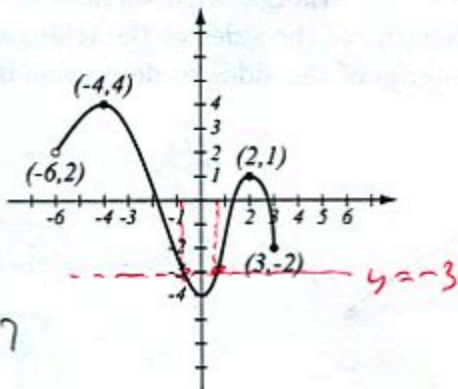
a) Find: $f(-4) = 4$ $f(4) = $ not defined

b) What is the domain of f? $(-6, 3]$

c) What is the range of f? $[-4, 4]$

d) What are the solutions of the equation $f(x) = -3$? $x = -1, 0.7$

(-4,4)   (-6,2)   (2,1)   (3,-2) $y = -3$

2. (12pts) Use your calculator to accurately sketch the graph of $f(x) = x^3 - 8\sqrt{x+2}$.

a) Draw the graph on paper and indicate units on the axes.

b) Find all the $x-$ and $y-$intercepts (accuracy: 6 decimal points).

c) State the domain of the function in interval notation.

10   5   -5   2.577081

b) x-int: 2.577081

y-int: -11.31371

c) Must have
$x + 2 \geq 0$
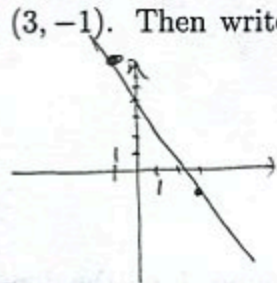$x \geq -2$ $[-2, \infty)$
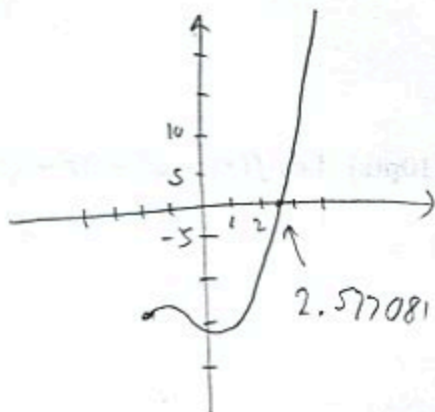
3. (5pts) Draw the line that passes through points $(-1, 7)$ and $(3, -1)$. Then write the equation of the line in form $y = mx + b$.

$m = \dfrac{-1 - 7}{3 - (-1)} = \dfrac{-8}{4} = -2$

$y - 7 = -2(x - (-1))$
$y - 7 = -2x - 2$
$y = -2x + 5$

4. (10pts) Find the equation of the line (in form $y = mx + b$) that is perpendicular to the line $3x + 2y = 8$ and has x-intercept 3. Draw both lines.
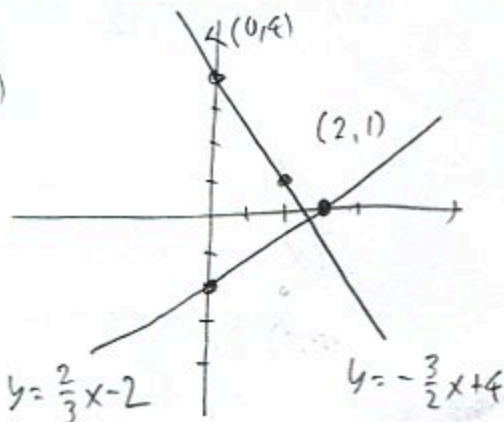
$3x + 2y = 8$
$2y = -3x + 8 \quad | \div 2$
$y = -\frac{3}{2}x + 4$
$m = -\frac{3}{2}$

Slope of perp. line
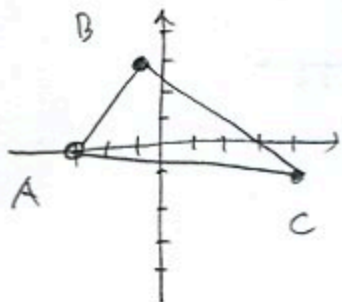is $\frac{2}{3}$, passes through $(3,0)$
$y - 0 = \frac{2}{3}(x - 3)$
$y = \frac{2}{3}x - 2$

$(0,4)$ $(2,1)$ $y = \frac{2}{3}x - 2$ $y = -\frac{3}{2}x + 4$

5. (8pts) Draw the triangle with vertices $A = (-3, 0)$, $B = (-1, 3)$ and $C = (4, -1)$.
a) Find the lengths of the sides of the triangle.
b) Use the lengths of the sides to determine if this is a right triangle.

a) $d(A, B) = \sqrt{(-1-(-3))^2 + (3-0)^2} = \sqrt{2^2 + 3^2} = \sqrt{13}$

$d(B, C) = \sqrt{(4-(-1))^2 + (-1-3)^2} = \sqrt{5^2 + (-4)^2} = \sqrt{41}$

$d(A, C) = \sqrt{(4-(-3))^2 + (-1-0)^2} = \sqrt{7^2 + (-1)^2} = \sqrt{50} \leftarrow$ largest

$\sqrt{13}^2 + \sqrt{41}^2 \stackrel{?}{=} \sqrt{50}^2$

$13 + 41 = 50$

not true, so triangle is not right.

6. (10pts) Let $f(x) = x^2 + 3x - \sqrt{x} + 1$. Find the following (simplify where appropriate).

$f(1) = 1^2 + 3 \cdot 1 - \sqrt{1} + 1 = 4$

$f(-4) = (-4)^2 + 3 \cdot (-4) - \sqrt{-4} + 1 = $ not defined

not defined

$f(a^2) = (a^2)^2 + 3a^2 - \sqrt{a^2} + 1$

$= a^4 + 3a^2 - a + 1$

$f(x-2) = (x-2)^2 + 3(x-2) - \sqrt{x-2} + 1$

$= x^2 - 2x - 2x + 4 + 3x - 6 - \sqrt{x-2} + 1$

$= x^2 - x - 1 + \sqrt{x-2}$

7. (6pts) Find the domain of the function below and write it using interval notation.

$f(x) = \dfrac{2x+1}{x^2 - 4x - 12}$

Can't have

$x^2 - 4x - 12 = 0$

$(x-6)(x+2) = 0$

$x - 6 = 0$ or $x + 2 = 0$

$x = 6, -2$

-2   6

$(-\infty, -2) \cup (-2, 6) \cup (6, \infty)$

8. (7pts) Solve and write the solution in interval notation.

$2x + 1 < 3$ or $3x - 5 > 10$

$2x < 2 \qquad 3x > 15$

$x < 1$ or $x > 5$ $\qquad (-\infty, 1) \cup (5, \infty)$

9. (8pts) A circle has center $(3, -2)$ and passes through the point $(2, 2)$.
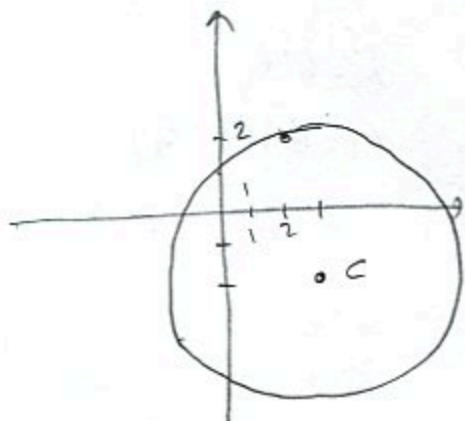a) Find the equation of the circle.
b) Draw the circle in the coordinate plane.

a) $r =$ distance from $(3, -2)$ to $(2, 2)$

$= \sqrt{(2-3)^2 + (2-(-2))^2} = \sqrt{(-1)^2 + 4^2} = \sqrt{17}$

$(x-3)^2 + (y-(-2))^2 = \sqrt{17}^2$

$(x-3)^2 + (y+2)^2 = 17$

10. (12pts) Two cell phone companies charge the following monthly fees for mobile data:
— U-Mobile charges \$5 per gigabyte of data.
— BSS charges \$20 for the first 8 gigabytes, and \$7 per gigabyte for data over 8 gigabytes. Assuming you always use more than 8 gigabytes of data, for which amounts of data is U-Mobile better? Solve as an inequality.

$x =$ no. of gigabytes used

U-Mobile cost: $5x$

BSS cost: $20 + 7(x-8)$

U-Mob. is cheaper

$5x < 20 + 7(x-8)$

$5x < 20 + 7x - 56$

$5x < 7x - 36$
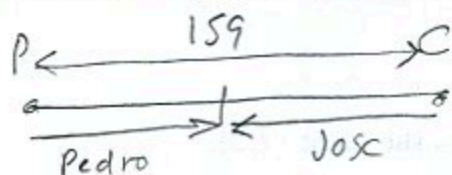
$5x + 36 < 7x$

$36 < 2x$

$18 < x$

For monthly usage above 18 gigabytes, U-Mobile is better

11. (14pts) A 159-mile road connects the Kentucky towns Princeton and Columbia. Pedro heads out from Princeton toward Columbia at 61mph. Jose heads out from Columbia toward Princeton 15 minutes later at 64mph. After a while, the friends meet on the road.
a) How long did each of them drive until they met?
b) How far from Princeton did they meet?

$P \xleftarrow{\quad\quad 159 \quad\quad} C$

Pedro   Jose

Jose was on rode $\frac{1}{4}$ less than Pedro

$d_1, 61, t \qquad d_2, 64, t-\frac{1}{4}$

$t = \frac{175}{125} = 1.4$ hrs

$d_1 = 61t$

$d_2 = 64(t-\frac{1}{4})$

a) Pedro drove 1.4 hours
Jose drove $1.4 - 0.25 = 1.15$ hours

$d_1 + d_2 = 159$

b) $d_1 = 61 \cdot 1.4 = 85.4$

$61t + 64(t-\frac{1}{4}) = 159$

$61t + 64t - 16 = 159 \quad | +16$

85.4 miles from Princeton

$125t = 175$

Bonus (10pts) Madison will invest some money into accounts bearing 3.5% and 4% simple interest. She plans to invest $1,000 more into the account bearing 4% interest than the account bearing 3.5% interest. If she wishes to have at least $193 in total interest after 1 year, what is the least she can invest in the account bearing 3.5% interest? Solve as an inequality.

$x =$ amount invested at 3.5%

$x + 1000 =$ amount invested at 4%

interest from 3.5%   interest from 4%

$\overline{0.035 \cdot x \cdot 1} + \overline{0.04 \cdot (x+1000) \cdot 1} \geq 193$

$0.035x + 0.04x + 40 \geq 193$

$0.075x \geq 153$

$x \geq \dfrac{153}{0.075} = 2040$

Madison needs to invest at least $2040 in the 3.5% account
(3040 in the 4% account)
to get at least $193 in interest.