Calculus 3 — Exam 3
MAT 309, Spring 2021 — D. Ivanšić

Name: _____

Show all your work!

1. (16pts) Let D be the region in the first quadrant bounded by the curves $y = \sqrt{x}$, $x = 0$ and $y = 2$.

a) Sketch the region D.

b) Set up $\iint_D \frac{1}{y^3 + 1}\, dA$ as iterated integrals in both orders of integration.

c) Evaluate the double integral using the easier order.

2. (12pts) Let D be the region that is under both curves $y = \sin x$ and $y = \cos x$ and above the x axis, and where $0 \leq x \leq \frac{\pi}{2}$. Set up $\iint_D x + y\, dA$, but do not evaluate the integral. Sketch the region of integration first.

3. (20pts) Use polar coordinates to find $\iint_D \frac{x}{x^2+y^2}\,dA$, if D is the region inside the circle $x^2+y^2=\frac{1}{4}$, and outside the cardioid $r=1+\cos\theta$. Sketch the region of integration first.

4. (18pts) Sketch the region E in the first octant $(x, y, z \geq 0)$ that is inside the cylinder $y^2+z^2=4$ and "behind" the plane $y=3x$. Then write the two iterated triple integrals that stand for $\iiint_E f\,dV$ which end in $dz\,dy\,dx$ and $dy\,dz\,dx$.

5. (20pts) Use cylindrical or spherical coordinates to evaluate $\iiint_E z\,dV$, if E is the region that is above the cone $z = \sqrt{3x^2 + 3y^2}$ and inside the sphere $x^2 + y^2 + z^2 = 9$. Sketch the region E.

6. (14pts) Use cylindrical coordinates to set up the integral for the volume of a spherical cap, the region inside the sphere $x^2 + y^2 + z^2 = a^2$ that is above the plane $z = b$, where $a > 0$ and $0 \leq b \leq a$. Do not evaluate the integral. Sketch the region E.

Bonus (10pts) Sketch the surfaces given by the equations:

$$z = \frac{1}{\sqrt{x^2 + y^2}} \qquad\qquad \rho = 1 + \sin\phi$$