Calculus 3 — Exam 2
MAT 309, Spring 2021 — D. Ivanšić

Name: ____________________

Show all your work!

1. (10pts) Let $f(x, y) = \sqrt{y - x^2}$.
a) Find the domain of f.
b) Sketch the contour map for the function, drawing level curves for levels $k = -1, 0, 1, 2$. Note the domain on the picture.
c) Suppose $f(x, y)$ is the temperature at point (x, y) and a heat-seeking insect (always moves in direction of greatest heat increase) starts at point $(1, 2)$. Sketch the path the insect will take and explain.

2. (16pts) Let $f(x, y) = xe^{x^3+y^3}$.
a) At point $(1, 0)$, find the directional derivative of f in the direction of $\langle -2, 1 \rangle$.
b) In what direction is the directional derivative the greatest, and what is its value?

3. (12pts) Consider the elliptical cone $y^2 + 3z^2 - x^2 = 0$.
a) Find the equation of the tangent plane to the cone at a generic point (x_0, y_0, z_0). Simplify the equation, keeping in mind that the point (x_0, y_0, z_0) satisfies the equation of the cone.
b) Show that the tangent plane always contains the origin.

4. (18pts) Let $U = \dfrac{\ln x}{xy}$, $x = \sqrt{st}$, $y = s^2 - t^2$. Use the chain rule to find $\dfrac{\partial U}{\partial s}$ when $s = 1$, $t = 2$.

5. (12pts) The range of a projectile fired at angle α with initial velocity v is given by $R = \dfrac{v^2 \sin(2\alpha)}{10}$ (R is in meters, v in meters per second, α in radians). Use differentials to estimate the change in range of a projectile fired at 40 m/s at angle $\frac{\pi}{6}$ if velocity is decreased by 0.2 meters per second, and angle is increased by 0.1 radian.

6. (12pts) Use implicit differentiation to find $\dfrac{\partial z}{\partial x}$ at the point $\left(0, \frac{\pi}{4}, \frac{\pi}{4}\right)$, if $\tan x + \tan y + \tan z = xyz + 2$.

7. (20pts) Find and classify the local extremes for $f(x, y) = 3x^2y + y^3 - 3x^2 - 3y^2$.

Bonus (10pts) Let $A = (0, 0)$, $B = (1, 0)$ and $C = (0, 2)$ and let d_A, d_B and d_C represent the distance from a point (x, y) to A, B and C, respectively. Find the absolute maximum and minimum of $d_A^2 + d_B^2 + d_C^2$ among all points (x, y) in the triangle ABC (edges are included).