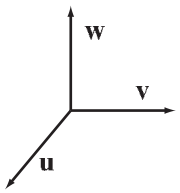
Calculus 3 — Exam 1
MAT 309, Spring 2021 — D. Ivanšić

Name: _____

Show all your work!

1. (11pts) Let $\mathbf{u} = \langle 1, 3, -1 \rangle$ and $\mathbf{v} = \langle 0, 2, 1 \rangle$.
a) Calculate $-2\mathbf{u}$, $3\mathbf{v} - 4\mathbf{u}$, and $\mathbf{u} \cdot \mathbf{v}$.
b) Find a vector of length $\sqrt{5}$ in direction of $\mathbf{u}$.
c) If θ is the angle between $\mathbf{u}$ and $\mathbf{v}$, find $\cos\theta$.

2. (12pts) In the picture, the vectors $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ are mutually perpendicular and all have length 3.
a) Draw the vector $\mathbf{u} - \mathbf{v}$ with its tail coinciding with the other tails.
b) Which is longer (if any): $\mathbf{u} \times \mathbf{v}$ or $\mathbf{u} \times (\mathbf{u} - \mathbf{v})$?
c) Draw the vector $\mathbf{w} \times (\mathbf{u} - \mathbf{v})$. Accurate length is not important.



3. (8pts) Draw the set in $\mathbf{R}^3$ described by:

$x^2 + y^2 + z^2 \geq 1$, $y = x$

4. (12pts) Find the equation of the plane that contains the lines given by parametric equations: $x = 1 + 2t$, $y = -2 - t$, $z = -3 + 4t$ and $x = 5 - t$, $y = -4 + 3t$, $z = 5 + t$. (These lines intersect — or they wouldn't determine a plane — but the point of intersection is not needed, so don't look for it.)

5. (16pts) This problem is about the surface $x^2 - 2y^2 + 5z^2 = 0$.
a) Identify and sketch the intersections of this surface with the coordinate planes.
b) Sketch the surface in 3D, with coordinate system visible.

6. (14pts) The curve $\mathbf{r}(t) = \langle 2\cos t, 2\sin t, \frac{1}{3}\sin(4t)\rangle$ is given, t any real number.
a) Sketch the curve in the coordinate system.
b) Find parametric equations of the tangent line to this curve when $t = \frac{\pi}{2}$ and sketch the tangent line.

7. (13pts) The points $A = (1, 3, -2)$ and $B = (4, -1, 3)$ are given.
a) Write parametric equations of the line segment AB.
b) Compute the length of the line segment using the parametrization and arc length formula.
c) Compare your answer in b) with the distance from A to B.

8. (14pts) An arrow is launched from ground level at a 45° angle with initial speed 50 meters per second.
a) Assuming gravity acts in the usual negative y-direction (let $g = 10$), find the vector function $\mathbf{r}(t)$ representing the position of the arrow.
b) Find the range of the arrow.
c) Find the maximum height the arrow reaches.

Bonus (10pts) Find the parametric equations of the line that is the intersection of the planes $x - y + 2z = 2$ and $x - y - 3z = 6$.