Spring '07/MAT 335/Exam 3 **Name:** *Show all your work.*

1. (4pts) By inspection, explain why the following sets of vectors cannot be bases for $\mathbf{R}^2$ and $\mathbf{R}^3$, respectively.

a) $\mathbf{v}_1 = (1, 1)$, $\mathbf{v}_2 = (-1, 2)$, $\mathbf{v}_3 = (0, 1)$

b) $\mathbf{v}_1 = (1, 2, 0)$, $\mathbf{v}_2 = (0, 2, 1)$, $\mathbf{v}_3 = (1, 0, -1)$

2. (5pts) Use matrix multiplication to find the matrix of the linear operator that is the composition of a rotation by 45° around the x axis, followed by a projection to the xy-plane.

3. (4pts) Find the standard matrix of the linear operator given by the equations below and determine whether it is a) one-to-one, or b) onto.

$$\begin{aligned} w_1 &= 5x_1 - 3x_2 \\ w_2 &= -x_1 + \tfrac{3}{5}x_2 \end{aligned}$$

4. (9pts) A matrix A is given below.
a) Find a basis for the row space of A.
b) Find a basis for the nullspace of A.
c) Verify that row(A)=null(A)$^{\perp}$ by showing that every basis vector for row(A) is orthogonal to every basis vector for null(A).

$$A = \begin{bmatrix} 1 & 3 & -2 & 0 \\ 2 & 6 & -5 & -2 \\ 0 & 0 & 5 & 10 \end{bmatrix}$$

5. (5pts) Let W be the subspace of $\mathbf{R}^3$ spanned by vectors $(2, 1, 4)$ and $(1, -1, 0)$. Find a basis for $W^{\perp}$.

6. (6pts) Let A be a 3×7 matrix. Answer the following and justify your answers.
a) What is the biggest rank(A) could be?
b) What is the smallest nullity(A) could be?
c) Give an example of a 3×7 matrix whose nullity is 5.

7. (4pts) Are the following vectors a basis for the subspace of $\mathbf{R}^5$ that they span?

$\mathbf{v}_1 = (*, *, *, *, 1)$, $\mathbf{v}_2 = (*, *, *, 1, 0)$, $\mathbf{v}_3 = (*, *, 1, 0, 0)$

8. (4pts) Complete the vector $(0, -1, 1)$ to a basis of $\mathbf{R}^3$. (That is, find additional vectors with which $(0, -1, 1)$ makes a basis.)

9. (9pts) Are the following statements true or false? Justify your answer by giving a logical argument or a counterexample.
a) If E is an elementary matrix, then A and EA have the same row space.
b) If A is a *nonzero* $m \times n$ matrix, then nullity$(A) \leq n - 1$.
c) For every 2×2 matrix A, row(A^T)=row$(A)^{\perp}$.

Bonus. (5pts) Let $\mathbf{v}_1 = (0, 3, -6, 5)$, $\mathbf{v}_2 = (0, 1, -2, 3)$. Write a linear system whose solution space is span$\{\mathbf{v}_1, \mathbf{v}_2\}$.