Calculus 1 — Exam 1 **MAT 250, Fall 2022 — D. Ivanšić**

Name: ____________________

Show all your work!

1. (16pts) Use the graph of the function to answer the following. Justify your answer if a limit does not exist.

$\lim_{x\to 5+} f(x) =$

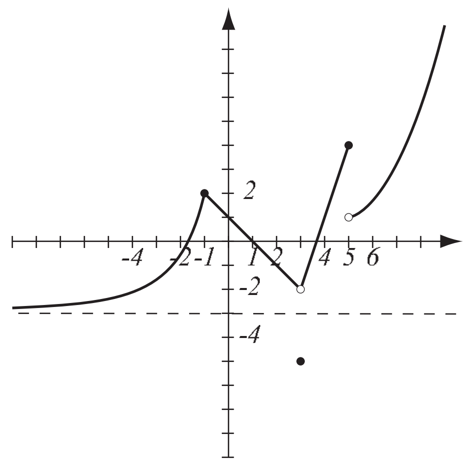
$\lim_{x\to 5^-} f(x) =$

$\lim_{x\to 5} f(x) =$

$\lim_{x\to -\infty} f(x) =$

$\lim_{x\to \infty} f(x) =$

$\lim_{x\to 3} f(x) =$



List points in domain of f where f is not continuous and justify why it is not continuous at those points.

2. (8pts) Draw the graph of a function $f(x)$ defined on the interval $(1, 6)$ which satisfies:

$\lim_{x\to 6^-} f(x) = -\infty$

$\lim_{x\to 1^+} f(x) = \infty$

f is discontinuous at $x = 3$,
continuous elsewhere

the equation $f(x) = 4$ has no solution

3. (10pts) Find $\lim_{x\to\infty} \dfrac{3+\cos x}{x^2}$. Use the theorem that rhymes with a dairy product people often put in sandwiches.

Find the following limits algebraically. Do not use the calculator.

4. (5pts) $\lim_{x\to 5} \dfrac{x^2 - 25}{x^2 + 2x - 35} =$

5. (7pts) $\lim_{x\to \infty} \dfrac{x + 2}{x^2 - 3x + 1} =$

6. (6pts) $\lim_{x\to 3^-} \dfrac{x - 4}{6 - 2x} =$

7. (7pts) $\lim_{x\to 4} \dfrac{x - 4}{\sqrt{x} - 2} =$

8. (7pts) $\lim_{x\to 0} \dfrac{\sin(2x)\sin x}{3x^2} =$

9. (14pts) The equation $2^x = 5 - x$ is given.

a) Use the Intermediate Value Theorem to show it has a solution in the interval $(0, 2)$.

b) Use your calculator to find an interval of length at most 0.01 that contains a solution of the equation. Then use the Intermediate Value Theorem to justify why your interval contains the solution.

10. (10pts) Consider the limit $\lim_{x\to 0}(1-2x)^{\frac{1}{x}}$. Use your calculator (don't forget parentheses) to estimate this limit with accuracy 3 decimal points. Write a table of values that will support your answer.

x	$(1-2x)^{\frac{1}{x}}$

x	$(1-2x)^{\frac{1}{x}}$

11. (10pts) Consider the function defined below.
a) Explain why the function is continuous on intervals $(-\infty, 1)$ and $(1, \infty)$
b) Is the function continuous at point $x = 1$?

$$f(x) = \begin{cases} 4x - 5, & \text{if } x \leq 1 \\ x^2 - 2x, & \text{if } x > 1. \end{cases}$$

Bonus. (10pts) Evaluate the function at the given x's. Then, based on the table, state what $\lim_{x \to 0} \dfrac{(x^4+2)^3 - 8}{x^4}$ appears to be. Explain any strange numbers you are getting.

x	$\dfrac{(x^4+2)^3 - 8}{x^4}$
0.1	
0.01	
0.001	
10^{-4}	
10^{-5}	
10^{-6}	

Calculus 1 — Exam 2
MAT 250, Fall 2022 — D. Ivanšić

Name: ______________________________

Show all your work!

Differentiate and simplify where appropriate:

1. (6pts) $\dfrac{d}{dx}\left(8x^5+\dfrac{9}{x^4}-\dfrac{1}{\sqrt[3]{x^2}}+\sqrt{a}\right)=$

2. (4pts) $\dfrac{d}{dx}\left(x^3\cos x\right)=$

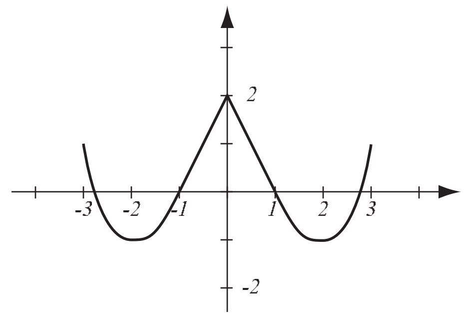
3. (6pts) $\dfrac{d}{ds}\dfrac{s^2+1}{s^2+4}=$

4. (6pts) $\dfrac{d}{d\theta}\dfrac{\sin^2\theta}{\cos\theta}=$

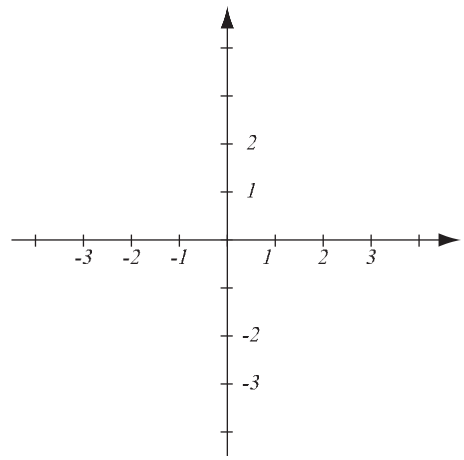
5. (6pts) $\dfrac{d}{dx}\sec\sqrt{x^2-3x+1}=$

6. (8pts) Let $y(x)=\sin(2x)$.
a) Write the first four derivatives of y.
b) Use the pattern you found in a) to find $y^{(43)}(x)$.

7. (10pts) The graph of the function $f(x)$ is shown at right.



a) Where is $f(x)$ not differentiable? Why?
b) Use the graph of $f(x)$ to draw an accurate graph of $f'(x)$.



8. (12pts) Let $f(x) = 2x^2 - 3x$.
a) Use the limit definition of the derivative to find the derivative of the function.
b) Check your answer by taking the derivative of f using differentiation rules.
c) Write the equation of the tangent line to the curve $y = f(x)$ at point $(1, -2)$.

9. (10pts) Let $g(x) = x^2 f(x)$ and $h(x) = f(f(x^2))$.
a) Find the general expressions for $g'(x)$ and $h'(x)$.
b) Use the table of values at right to find $g'(2)$ and $h'(2)$.

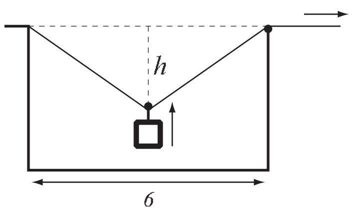
x	1	2	3	4
$f(x)$	-1	-3	3	1
$f'(x)$	-3	1	5	-1

10. (7pts) A ball thrown upwards has position given by the formula $s(t) = -5t^2 + 30t$.
a) Write the formula for the velocity of the ball at time t.
b) When does the ball reach its maximum height?
c) What is the maximal height of the ball?

11. (11pts) Use implicit differentiation to find y'.

$$\tan(xy) = \frac{x^2}{y} + y^2$$

12. (14pts) A safe is raised using the pulley system shown, with rope pulled at right at speed 0.1 meters per second, while it is anchored at left. How fast is the distance to the top h changing when the middle pulley is 2 meters from the top? The rope shortens evenly on both sides of the safe so it stays in the center.



Bonus. (10pts) Let $f(x) = \dfrac{1}{x^4}$. Use the limit definition of the derivative to find the derivative of the function and check it against the known result.

Calculus 1 — Exam 3
MAT 250, Fall 2022 — D. Ivanšić

Name: ______________________________

Show all your work!

Differentiate and simplify where appropriate:

1. (5pts) $\dfrac{d}{dx}\, e^{x^2-x+3} =$

2. (6pts) $\dfrac{d}{dx} \left(\sqrt{x} - \dfrac{1}{\sqrt{x}}\right) e^x =$

3. (6pts) $\dfrac{d}{dt}\, \dfrac{\arctan t}{t^2} =$

4. (7pts) $\dfrac{d}{dx}\, \ln \left(\dfrac{x+2}{x-2}\right)^3$

5. (7pts) $\dfrac{d}{d\theta}\, \ln(\sec\theta + \tan\theta) =$

6. (9pts) Use logarithmic differentiation to find the derivative of $y = (\sin x)^{\cos x}$.

Find the limits algebraically. Graphs of basic functions will help, as will L'Hospital's rule, where appropriate.

7. (2pts) $\lim_{x \to -\infty} e^{3x} =$

8. (7pts) $\lim_{x \to \infty} \arctan\left(\frac{x^2 + 5x + 1}{x + 7}\right) =$

9. (6pts) $\lim_{x \to \infty} \frac{x^2}{2^x} =$

10. (9pts) $\lim_{x \to 0} \frac{\tan x - x}{x^3} =$

11. (8pts) $\lim_{x \to 0+} x^x =$

12. (12pts) Let $f(x) = \ln x$.
a) Write the linearization of $f(x)$ at $a = 1$.
b) Use the linearization to estimate $\ln 1.2$.
c) In the same coordinate system, draw rough graphs of the function and the linearization and determine if the estimate overshoots or undershoots $\ln 1.2$.

13. (9pts) In a right triangle, the hypothenuse is known to be 5 inches. One of the angles is measured to be $\frac{\pi}{6}$ radians, with maximum error 0.1 radians. Use differentials to estimate the maximum possible error and relative error when computing the length of the side adjacent to the angle.

14. (7pts) Let $f(x) = x^3 - x$. Use the theorem on derivatives of inverses to find $(f^{-1})'(6)$.

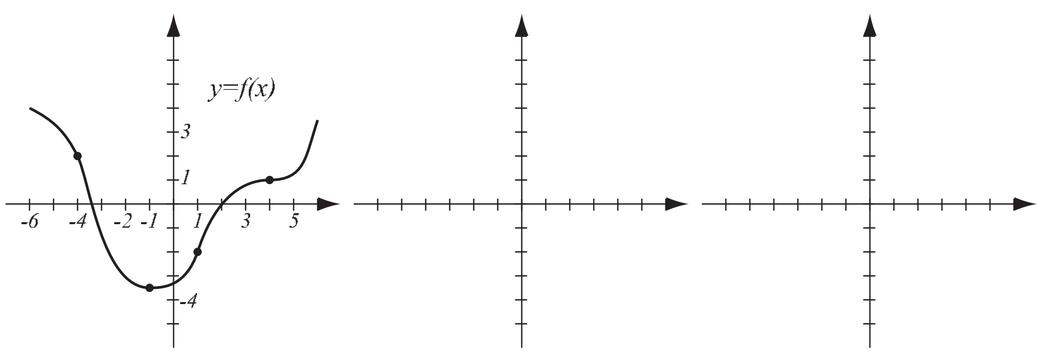
Bonus. (10pts) Let $f(x) = x^n$, $x \geq 0$, where n is a positive integer. We have justified the rule for the derivative of f using the definition by computing a limit. Use the derivative of f and either the theorem on derivatives of inverses, or implicit differentiation, to justify the rule for the derivative of $\sqrt[n]{x}$.

Calculus 1 — Exam 4 **MAT 250, Fall 2022 — D. Ivanšić**	**Name:**__________________________ *Show all your work!*

1. (30pts) Let $f(x) = \dfrac{x^2}{x^2+9}$. Draw an accurate graph of f by following the guidelines.
a) Find the intervals of increase and decrease, and local extremes.
b) Find the intervals of concavity and points of inflection.
c) Find $\lim\limits_{x\to\infty} f(x)$ and $\lim\limits_{x\to-\infty} f(x)$.
d) Use information from a)–c) to sketch the graph.

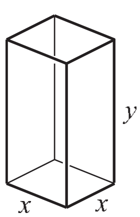
2. (16pts) Let $f(x) = x^2 e^x$. Find the absolute minimum and maximum values of f on the interval $[-4, 1]$.

3. (14pts) The graph of f is given. Use it to draw the graphs of f' and f'' in the coordinate systems provided. Pay attention to increasingness, decreasingness and concavity of f. The relevant special points have been highlighted.



4. (18pts) Consider $f(x) = \sin^2 x - \cos x$ on the interval $[0, 2\pi]$.
a) Verify that the function satisfies the assumptions of the Mean Value Theorem.
b) Find all numbers c that satisfy the conclusion of the Mean Value Theorem.

5. (22pts) A box with a square base uses 50 square inches of material (that is its surface area, including top and bottom). Find the dimensions x, y of the box that give the maximal possible volume of the box.



Bonus. (10pts) Let $f(x) = \frac{5}{9}x^{\frac{9}{5}} - \frac{15}{4}x^{\frac{4}{5}}$. Note domain is all real numbers.
a) Find the intervals of increase and decrease, and local extremes.
b) Find the intervals of concavity and points of inflection.
c) Use information from a) and b) to sketch the graph.

Calculus 1 — Exam 5
MAT 250, Fall 2022 — D. Ivanšić

Name: ______________________________

Show all your work!

Find the following antiderivatives or definite integrals.

1. (3pts) $\int \sqrt[5]{x^2}\,dx =$

2. (3pts) $\int \cos\left(3x+\frac{\pi}{2}\right)\,dx =$

3. (6pts) $\int \sqrt{t}(t^2-3t)\,dt =$

4. (4pts) $\int_1^e \frac{1}{x}\,dx =$

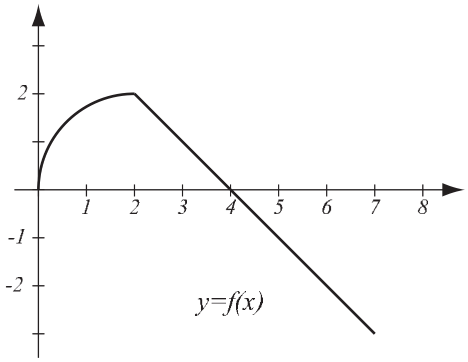
5. (7pts) $\int_0^{\frac{\pi}{6}} \frac{1+\cos^2\theta}{\cos^2\theta}\,d\theta =$

6. (6pts) Find $f(x)$ if $f'(x)=\frac{4}{\sqrt{1-x^2}}$ and $f\left(\frac{1}{2}\right)=3$.

7. (15pts) The function $f(x) = 4 - x^2$ is given on the interval $[0, 3]$.
a) Write the Riemann sum M_6 for this function with six subintervals, taking sample points to be midpoints. Do not evaluate the expression.
b) Illustrate with a diagram, where appropriate rectangles are clearly visible. What does M_6 represent?

8. (13pts) Find $\int_0^3 x - 1\,dx$ in two ways (they'd better give you the same answer!):
a) Using the "area" interpretation of the integral. Draw a picture.
b) Using the Evaluation Theorem.

9. (10pts) The graph of a function f, consisting of lines and parts of circles, is shown. Evaluate the integrals.

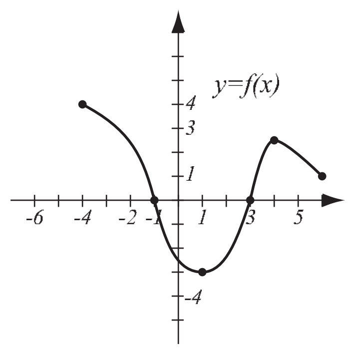


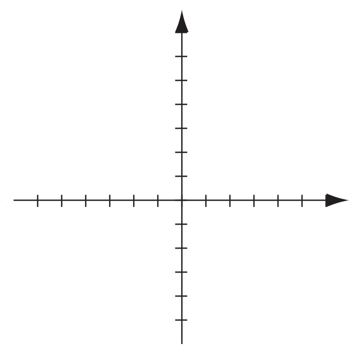
$\int_0^2 f(x)\,dx =$

$\int_2^7 f(x)\,dx =$

$\int_0^7 f(x)\,dx =$

10. (8pts) Pictured is the graph of $f(x)$. Sketch the graph of the antiderivative $F(x)$ of $f(x)$, if it is known that $F(-4) = 0$. Relevant points have been highlighted.





11. (7pts) Use the inequality $m(b-a) \leq \int_a^b f(x)\,dx \leq M(b-a)$, where $m \leq f(x) \leq M$ on $[a,b]$, to give an estimate of $\int_{-4}^{-2} f(x)\,dx$, where f is the function pictured in the preceding problem.

12. (6pts) Write using sigma notation:

$$\frac{2}{x^4} + \frac{3}{x^6} + \frac{4}{x^8} + \cdots + \frac{9}{x^{18}} =$$

13. (12pts) Toxic sludge is being deposited into a collection pool at rate $6\sqrt{t} + 3t$ cubic meters per hour.
a) Use the Net Change Theorem to find how much sludge was added from $t = 0$ to $t = 4$ hours.
b) If at time $t = 0$ there were 12 cubic meters of sludge in the pool, how much is there at $t = 4$ hours?

Bonus. (10pts) Recall that $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$. Use this formula to evaluate the sum below. (Note that it does not start with 1, how do you handle this?)

$$\sum_{i=4}^{n} (7 + 2i)$$

Calculus 1 — Final Exam
MAT 250, Fall 2022 — D. Ivanšić

Name: ____________________

Show all your work!

1. (16pts) Use the graph of the function to answer the following. Justify your answer if a limit does not exist.

$\lim_{x\to -3^+} f(x) =$

$\lim_{x\to 2-} f(x) =$

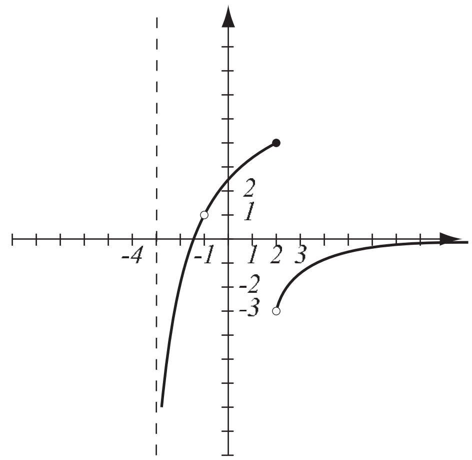
$\lim_{x\to 2+} f(x) =$

$\lim_{x\to 2} f(x) =$

$\lim_{x\to -1} f(x) =$

$\lim_{x\to \infty} f(x) =$

List points where f is not continuous and explain why.



Find the following limits algebraically. Do not use L'Hospital's rule.

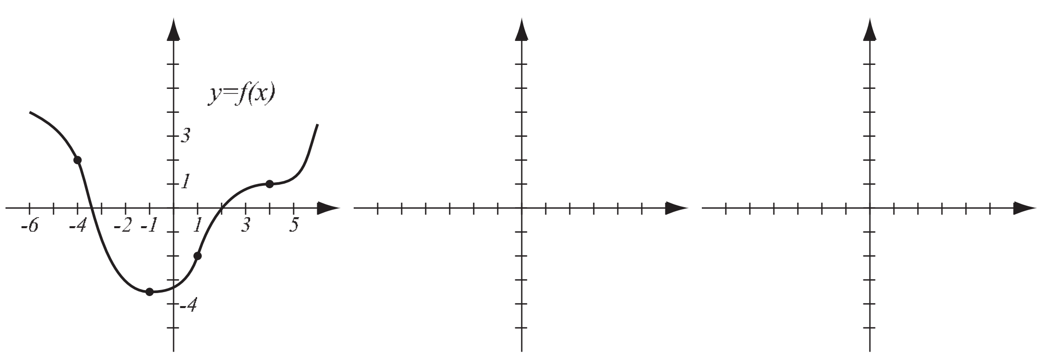
2. (6pts) $\lim_{x\to 2-} \frac{x+7}{3x-6} =$

3. (5pts) $\lim_{x\to 2} \frac{x^2+2x-8}{x^2-9x+14} =$

4. (6pts) The equation $x^3 - 2x^2 + x = 5$ is given. Use the Intermediate Value Theorem to show it has a solution.

5. (12pts) Let $f(x) = \sin x$.
a) Find the equation of the tangent line to the graph of f where $x = 0$.
b) The tangent line is the linearization of the function at $x = 0$. Use the linearization to estimate $\sin 0.2$.
c) In the same coordinate system, draw graphs of the function and the linearization and determine if the estimate overshoots or undershoots $\sin 0.2$.

6. (12pts) The graph of f is given. Use it to draw the graphs of f' and f'' in the coordinate systems provided. Pay attention to increasingness, decreasingness and concavity of f. The relevant special points have been highlighted.



7. (26pts) Let $f(x) = \dfrac{4 - x^2}{x^2 + 9}$. Draw an accurate graph of f by following the guidelines.

a) Find the intervals of increase and decrease, and local extremes.

b) Find the intervals of concavity and points of inflection.

c) Find $\lim_{x \to \infty} f(x)$ and $\lim_{x \to -\infty} f(x)$.

d) Use information from a)–c) to sketch the graph.

8. (10pts) Let $f(x) = \dfrac{x^2}{e^x}$. Find the absolute minimum and maximum values of f on the interval $[-1, 3]$.

9. (6pts) Find $f(x)$ if $f'(x) = \sqrt[4]{x^3} + \dfrac{1}{x}$ and $f(1) = 3$.

10. (11pts) Consider the integral $\displaystyle\int_0^3 4 - x^2\,dx$.

a) Use a picture and the "area" interpretation of the integral to determine whether this integral is positive or negative.

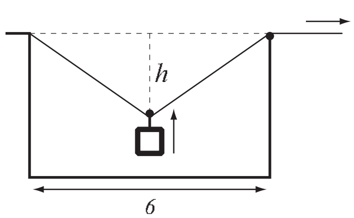
b) Use the Evaluation Theorem to find the integral and verify your conclusion from a).

11. (12pts) Toxic sludge is being deposited into a collection pool at rate $3\sqrt{t} + t$ cubic meters per hour.

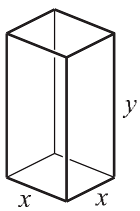
a) Use the Net Change Theorem to find how much sludge was added from $t = 0$ to $t = 4$ hours.

b) If at time $t = 0$ there were 12 cubic meters of sludge in the pool, how much is there at $t = 4$ hours?

12. (12pts) A safe is raised using the pulley system shown, with rope pulled at right at speed 0.3 meters per second, while it is anchored at left. How fast is the distance to the top h changing when the middle pulley is 1 meter from the top? The rope shortens evenly on both sides of the safe so it stays in the center.



13. (16pts) A box with a square base uses 98 square inches of material (that is its surface area, including top and bottom). Find the dimensions x, y of the box that give the maximal possible volume of the box. Don't forget to check it is a maximum.



Bonus. (10pts) Recall that $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$. Use this formula to evaluate the sum below. (Note that it does not start with 1, how do you handle this?)

$$\sum_{i=4}^{n}(7+2i)$$