Find the following antiderivatives.

1. (3pts) $\displaystyle \int \sqrt[7]{x^3}\, dx =$

2. (3pts) $\displaystyle \int \frac{5}{\sqrt{1-x^2}}\, dx =$

3. (3pts) $\displaystyle \int e^{4x+1}\, dx =$

4. (7pts) $\displaystyle \int \frac{s^3 + s}{\sqrt{s}}\, ds =$

5. (7pts) Find $f(x)$ if $\displaystyle f'(x) = \frac{3}{1+x^2} + \frac{1}{x}$ and $f(1) = 5$.

6. (6pts) Write using sigma notation:

$$\frac{5}{6} + \frac{7}{8} + \frac{9}{10} + \cdots + \frac{17}{18} =$$

7. (15pts) The function $f(x) = \sin x$ is given on the interval $\left[-\frac{\pi}{4}, \frac{\pi}{2}\right]$.
a) Write the Riemann sum R_6 for this function with six subintervals, taking sample points to be right endpoints. Do not evaluate the expression.
b) Illustrate with a diagram, where appropriate rectangles are clearly visible. What does R_6 represent?

8. (13pts) Find $\int_{-2}^{2} 3 - x \, dx$ in two ways (they'd better give you the same answer!):
a) Using the "area" interpretation of the integral. Draw a picture.
b) Using the Evaluation Theorem.

9. (7pts) The graph of a function f, consisting of lines and parts of circles, is shown. Evaluate the integrals.
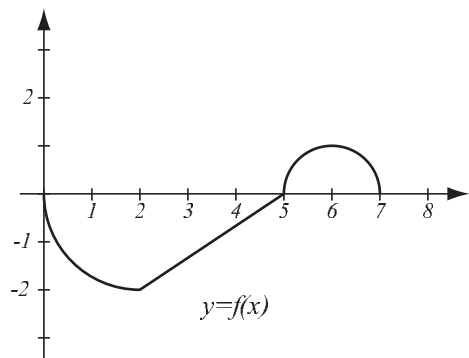
$\int_2^5 f(x)\,dx =$

$\int_5^7 f(x)\,dx =$

$\int_0^7 f(x)\,dx =$

Use the substitution rule in the following integrals:

10. (8pts) $\displaystyle\int \frac{3x^2 + 4x}{(x^3 + 2x^2 + 7)^2}\,dx =$

11. (10pts) $\displaystyle\int_0^{\ln 5} \frac{e^x}{\sqrt{4 + e^x}}\,dx =$

12. (8pts) $\displaystyle\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} (\sin^2 x - 3\sin x + 3)\cos x\,dx =$

13. (10pts) A ball traveling upwards has speed $v(t) = 27 - 10t$ meters per second..
a) Use the Net Change Theorem to find by how much the height of the ball has changed from $t = 0$ to $t = 3$.
b) If at time $t = 0$ the ball was at height 18 meters, at what height is it at $t = 3$?

Bonus. (10pts) Justify the following statements with pictures.
a) If f is even, then $\displaystyle\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$
b) If f is odd, then $\displaystyle\int_{-a}^{a} f(x)\,dx = 0$