

Calculus 1 — Exam 3
MAT 250, Fall 2019 — D. Ivanišić

Name: _____
Show all your work!

Differentiate and simplify where appropriate:

1. (5pts) $\frac{d}{dx} \ln(3x^2 - 5x + 2) =$

2. (6pts) $\frac{d}{dx} (x^{\frac{3}{2}} - 4x^{\frac{1}{2}})e^x =$

3. (6pts) $\frac{d}{du} \frac{\ln u}{u^2} =$

4. (7pts) $\frac{d}{dx} \arctan \sqrt{x^2 - 1} =$

5. (7pts) $\frac{d}{d\theta} \log_4 \frac{1 - \sec \theta}{1 + \sec \theta} =$

6. (10pts) Use logarithmic differentiation to find the derivative of $y = (x^2 - 1)^{x^2}$.

Find the limits algebraically. Graphs of basic functions will help, as will L'Hospital's rule, where appropriate.

7. (2pts) $\lim_{x \rightarrow 0^+} \log_3 x =$

8. (7pts) $\lim_{x \rightarrow \infty} \arccos \left(\frac{x+4}{x^2-3x+12} \right) =$

9. (6pts) $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} =$

10. (9pts) $\lim_{x \rightarrow \infty} \frac{x^{\frac{3}{2}}}{1.1^x} =$

11. (8pts) $\lim_{x \rightarrow 0} (1 - 2x)^{\frac{1}{\sin x}} =$

12. (10pts) Let $f(x) = \sqrt{x}$.

a) Write the linearization of $f(x)$ at $a = 16$.

b) Use the linearization to estimate $\sqrt{17}$ and compare to the calculator value of 4.123106.

13. (10pts) A cube is measured to have side length of 5 centimeters, with maximum error 2 millimeters. Use differentials to estimate the maximum possible error, the relative error and the percentage error when computing the volume of the cube.

14. (7pts) The table of values of $f(x)$ and $f'(x)$ is given at right. Use the theorem on derivatives of inverses to find $(f^{-1})'(1)$.

x	1	2	3	4
$f(x)$	4	1	0	-1
$f'(x)$	-2	-3	-4	-1

Bonus. (10pts) The function $f(x) = x^2 + 4x - 7$ is one-to-one on the domain $(-\infty, -2]$.

a) Use either the quadratic formula or completion of squares to find $f^{-1}(x)$.

b) Use the theorem on derivatives on inverses to find $(f^{-1})'(x)$ and compare it with the derivative that you get from the formula you find in a).