Calculus 1 — Exam 1
MAT 250, Fall 2019 — D. Ivanšić

Name: _____

Show all your work!

1. (16pts) Use the graph of the function to answer the following. Justify your answer if a limit does not exist.

$\lim_{x\to -2+} f(x) =$

$\lim_{x\to -2^-} f(x) =$

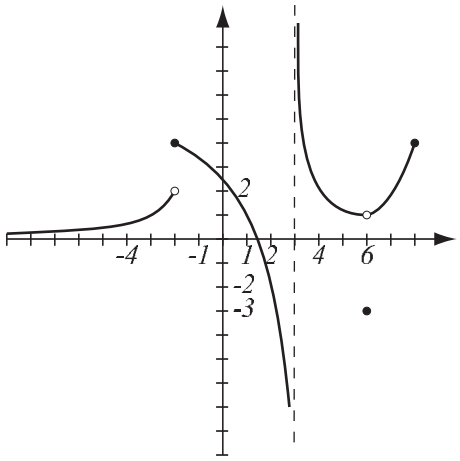
$\lim_{x\to -2} f(x) =$

$\lim_{x\to -\infty} f(x) =$

$\lim_{x\to 6} f(x) =$

$\lim_{x\to 3} f(x) =$

List points in domain of f where f is not continuous and justify why it is not continuous at those points.



2. (6pts) Let $\lim_{x\to 5} f(x) = 3$ and $\lim_{x\to 5} g(x) = 1$. Use limit laws to find the limit below and show each step.

$$\lim_{x\to 5} \frac{xf(x) - 9g(x)}{f(x)^2 - 3} =$$

3. (10pts) Find $\lim_{x\to 0} x^4 \cdot (1 + \sin\left(\frac{1}{x}\right))^2$. Use the theorem that rhymes with insects that you might find on dogs and cats.

Find the following limits algebraically. Do not use the calculator.

4. (7pts) $\lim_{x\to\infty} \dfrac{x^2-5x+7}{x+4} =$

5. (5pts) $\lim_{x\to 7} \dfrac{x^2-6x-7}{x^2-11x+28} =$

6. (7pts) $\lim_{x\to 5} \dfrac{x^2-25}{\sqrt{x}-\sqrt{5}} =$

7. (6pts) $\lim_{x\to -3^-} \dfrac{x}{x+3} =$

8. (7pts) $\lim_{x\to 1^+} \left(\dfrac{1}{x-1} - \dfrac{2}{x^2-1}\right) =$

9. (8pts) The equation $x^3 - x^2 + x = \sqrt{x} + 1$ is given. Use the Intermediate Value Theorem to show it has a solution in the interval $(0, 4)$.

10. (8pts) Explain in an intuitive way why the Intermediate Value Theorem is true on this example: Let f be a continuous function defined on the interval $[2, 5]$, and let $f(2) = -1$ and $f(5) = 4$. Justify graphically why there has to be a number c in the interval $(2, 5)$ so that $f(c) = 3$. (You need a picture and a nice sentence.)

11. (10pts) Consider the limit $\lim_{x \to 2} \dfrac{2^x - 4}{x - 2}$. Use your calculator (don't forget parentheses) to estimate this limit with accuracy 3 decimal points. Write a table of values that will support your answer.

x	$\dfrac{2^x - 4}{x - 2}$

x	$\dfrac{2^x - 4}{x - 2}$

12. (10pts) Is the function defined below continuous? Justify.

$$f(x) = \begin{cases} \dfrac{\sin(2x-6)}{x-3}, & \text{if } x \neq 3 \\ \\ 1, & \text{if } x = 3. \end{cases}$$

Bonus. (10pts) Show by example that the conclusion of the Intermediate Value Theorem is not true if the function is not continuous. Draw a function defined on the interval $[2, 5]$ for which $f(2) = -1$ and $f(5) = 4$. but there is no number c in the interval $(2, 5)$ so that $f(c) = 3$.