Calculus 1 — Exam 1
MAT 250, Fall 2022 — D. Ivanšić

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Show all your work!

1. (16pts) Use the graph of the function to answer the following. Justify your answer if a limit does not exist.

$\lim_{x\to 5+} f(x) =$ 1

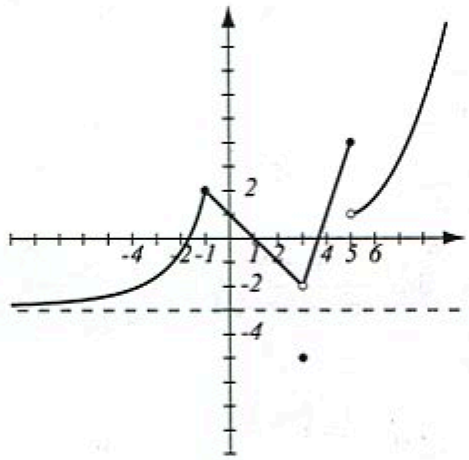
$\lim_{x\to 5^-} f(x) =$ 4

$\lim_{x\to 5} f(x) =$ dne, since one-sided limits are different

$\lim_{x\to -\infty} f(x) = -3$

$\lim_{x\to \infty} f(x) = \infty$

$\lim_{x\to 3} f(x) = -2$



List points in domain of f where f is not continuous and justify why it is not continuous at those points.

Not cont. at $x=5$, $\lim_{x\to 5} f(x)$ dne

$x=3$ $\lim_{x\to 3} f(x) \neq f(3)$

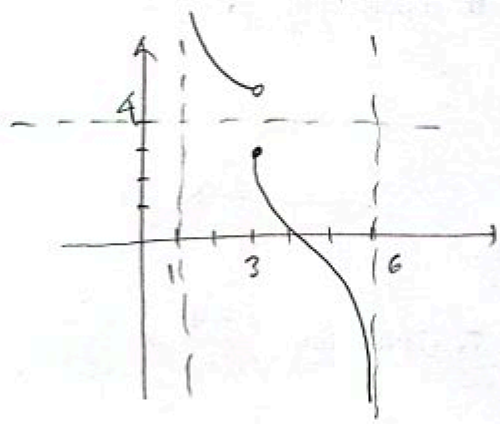
2. (8pts) Draw the graph of a function $f(x)$ defined on the interval $(1,6)$ which satisfies:

$\lim_{x\to 6^-} f(x) = -\infty$

$\lim_{x\to 1^+} f(x) = \infty$

f is discontinuous at $x = 3$, continuous elsewhere

the equation $f(x) = 4$ has no solution



For example

3. (10pts) Find $\lim_{x\to\infty} \frac{3+\cos x}{x^2}$. Use the theorem that rhymes with a dairy product people often put in sandwiches.

$-1 \le \cos x \le 1$

$2 \le 3+\cos x \le 4$

$\frac{2}{x^2} \le \frac{3+\cos x}{x^2} \le \frac{4}{x^2}$

$\lim_{x\to\infty} \frac{2}{x^2} = \frac{2}{\infty} = 0$

$\lim_{x\to\infty} \frac{4}{x^2} = \frac{4}{\infty} = 0$

equal, so by squeeze theorem

$\lim_{x\to\infty} \frac{3+\cos x}{x^2} = 0$

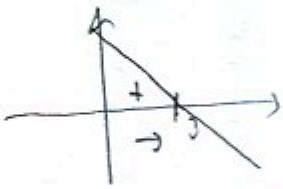
Find the following limits algebraically. Do not use the calculator.

4. (5pts) $\lim_{x\to 5} \frac{x^2-25}{x^2+2x-35} = \lim_{x\to 5} \frac{\cancel{(x-5)}(x+5)}{\cancel{(x-5)}(x+7)} = \lim_{x\to 5} \frac{x+5}{x+7} = \frac{10}{12} = \frac{5}{6}$

5. (7pts) $\lim_{x\to \infty} \frac{x+2}{x^2-3x+1} = \lim_{x\to\infty} \frac{x\left(1+\frac{2}{x}\right)}{x^2\left(1-\frac{3}{x}+\frac{1}{x^2}\right)} = \lim_{x\to\infty} \frac{1}{x} \frac{\left(1+\frac{2}{x}\right)}{\left(1-\frac{3}{x}+\frac{1}{x^2}\right)}$

$$= \frac{1}{\infty} \cdot \frac{1+\frac{2}{\infty}}{1-\frac{3}{\infty}+\frac{1}{\infty}} = 0 \cdot \frac{1+0}{1-0+0} = 0 \cdot 1 = 0$$

6. (6pts) $\lim_{x\to 3^-} \frac{x-4}{6-2x} = \frac{3-4}{6-6} = \frac{-1}{0^+} = -1 \cdot \infty = -\infty$



For $x<3$

$6-2x>0$

7. (7pts) $\lim_{x\to 4} \frac{x-4}{\sqrt{x}-2} = \frac{\sqrt{x}+2}{\sqrt{x}+2} = \lim_{x\to 4} \frac{(x-4)(\sqrt{x}+2)}{\sqrt{x}^2-2^2} = \lim_{x\to 4} \frac{\cancel{(x-4)}(\sqrt{x}+2)}{\cancel{x-4}}$

$$= \sqrt{4}+2 = 4$$

8. (7pts) $\lim_{x\to 0} \frac{\sin(2x)\sin x}{3x^2} = \lim_{x\to 0} \frac{1}{3} \frac{\sin(2x)}{x} \cdot \frac{\sin x}{x} = \lim_{x\to 0} \frac{2}{3} \frac{\sin(2x)}{2x} \cdot \frac{\sin x}{x}$

$$= \frac{2}{3} \cdot 1 \cdot 1 = \frac{2}{3}$$

9. (14pts) The equation $2^x = 5 - x$ is given.
a) Use the Intermediate Value Theorem to show it has a solution in the interval $(0, 2)$.
b) Use your calculator to find an interval of length at most 0.01 that contains a solution of the equation. Then use the Intermediate Value Theorem to justify why your interval contains the solution.

a) $\underbrace{2^x + x - 5}_{f(x),\ \text{continuous}} = 0$

$f(0) = 2^0 + 0 - 5 = -4$

$f(2) = 2^2 + 2 - 5 = 1$

Since $-4 < 0 < 1$, by Intermediate Value Theorem, there is a c s.t. $f(c) = 0$

b) From calculator we see that

$f(1.71) = -0.0184$

$f(1.72) = 0.01436$

Since $-0.0184 < 0 < 0.01436$, by IVT there is a c in $(1.71, 1.72)$ such that $f(c) = 0$.

10. (10pts) Consider the limit $\lim_{x \to 0} (1 - 2x)^{\frac{1}{x}}$. Use your calculator (don't forget parentheses) to estimate this limit with accuracy 3 decimal points. Write a table of values that will support your answer.

x	$(1-2x)^{\frac{1}{x}}$
0.1	0.10737
0.01	0.13262
0.001	0.13506
10^{-4}	0.13531
10^{-5}	0.13533
10^{-6}	0.13534

x	$(1-2x)^{\frac{1}{x}}$
-0.1	0.16151
-0.01	0.13803
-0.001	0.13561
-10^{-4}	0.13536
-10^{-5}	0.13534
-10^{-6}	0.13534

From table, it appears that $\lim_{x \to 0} (1-2x)^{\frac{1}{x}} \approx 0.135$

11. (10pts) Consider the function defined below.
a) Explain why the function is continuous on intervals $(-\infty, 1)$ and $(1, \infty)$
b) Is the function continuous at point $x = 1$?

$$f(x) = \begin{cases} 4x - 5, & \text{if } x \leq 1 \\ x^2 - 2x, & \text{if } x > 1. \end{cases}$$

a) $4x-5$ and x^2-2x are polynomials, so are continuous on all real numbers. Therefore f is continuous where these functions apply for f, on $(-\infty,1)$ and $(1,\infty)$ respectively

b) $\lim_{x\to1^-} f(x) = \lim_{x\to1^-}(4x-5) = 4\cdot1-5 = -1$

$\lim_{x\to1^+} f(x) = \lim_{x\to1^+}(x^2-2x) = 1^2-2\cdot1 = -1$

Thus, $\lim_{x\to1} f(x) = -1$

and $f(1) = -1$

So f is continuous at 1

Bonus. (10pts) Evaluate the function at the given x's. Then, based on the table, state what $\lim_{x\to0}\dfrac{(x^4+2)^3-8}{x^4}$ appears to be. Explain any strange numbers you are getting.

x	$\dfrac{(x^4+2)^3-8}{x^4}$
0.1	12.001
0.01	12
0.001	12
10^{-4}	0
10^{-5}	0
10^{-6}	0

It appears that $\lim_{x\to0} f(x) = 12$.

For smaller values of x, we run into calculator limitations. Because it carries only a certain number of significant digits, addition x^4+2 results in 2 for small x:

$(10^{-4})^4+2 = 10^{-16}+2$. To the calculator, $2+10^{-16} \approx 2$, because it carries only some 13 significant digits, then to calculator

$$\frac{(10^{-16}+2)^3-8}{10^{-16}} \approx \frac{2^3-8}{10^{-16}} = \frac{0}{10^{-16}} = 0$$

[0]Total points: 100