Mathematical Concepts — Joysheet 7 MAT 117, Fall 2022 — D. Ivanšić

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Covers: 11.7, 11.8 *Show all your work!*

1. (14pts) Motorcycle racer Whelan knows that racing is not without risks of bodily injury. Surveying data, he finds that in one race, the likelihood that a rider emerges unscathed is 81%, lightly injured 15%, severely injured 4%. Assume Whelan goes to two races, whose injury outcomes are independent of each other. What is the probability that
a) Whelan is unscathed in the first race and severely injured in the second?
b) Whelan is lightly injured in one race and unscathed in the other?
c) Whelan is injured in at least one race?

a) P(unscathed 1st AND sev. inj. 2nd) = P(unscathed 1st) · P(sev. inj. 2nd)
$= 0.81 \cdot 0.04 = 0.0324$

mutually exclusive

b) P(l. inj. in one race, unsc. in other) = P((l. inj 1st AND unsc. 2nD) OR (unsc. 1st AND l. inj. 2nd)) =
= P(l. inj. 1st AND unsc. 2nd) + P(unsc. 1st AND l. inj. 2nd) =
= P(l. inj. 1st) · P(unsc. 2nd) + P(unsc. 1st) · P(l. inj. 2nd) $= 0.15 \cdot 0.81 + 0.81 \cdot 0.15 = 0.243$

c) P(inj. in at least one race) = 1 − P(unscathed in both) = 1 − P(unsc. 1st AND unsc. 2nd)
= 1 − P(unsc. 1st) · P(unsc. 2nd) $= 1 - 0.81 \cdot 0.81 = 0.3439$

2. (14pts) Three Coke cans are picked at random in succession from a trunk with 9 regular Cokes, 6 diet Cokes, 4 vanilla Cokes and 5 cherry Cokes. What is the probability that:
a) The second can is a cherry Coke, given that the first one was diet Coke?
b) The first two cans are regular Cokes and the third is a cherry Coke?
c) All three are not regular Cokes?
d) At least one is a vanilla Coke?

$9+6+4+5 = 24$ cans

a) P(cherry 2nd | diet 1st) $= \frac{5}{23}$

b) P(reg 1st AND reg. 2nd AND cherry 3rd)
= P(reg 1st) · P(reg. 2nd | reg. 1st) · P(cherry 3rd | reg 1st AND reg. 2nd)
$= \frac{9}{24} \cdot \frac{8}{23} \cdot \frac{5}{22} = \frac{3}{\cancel{8}} \cdot \frac{\cancel{8}}{23} \cdot \frac{5}{22} = \frac{15}{506}$

c) P(not reg 1st AND not reg 2nd AND not reg 3rd)
= P(not reg. 1st) · P(not reg. 2nd | not reg. 1st) · P(not reg. 3rd | not reg. 1st AND not reg. 2nd)
$= \frac{15}{24} \cdot \frac{\cancel{14}\,7}{23} \cdot \frac{13}{\cancel{22}\,11} = \frac{5}{8} \cdot \frac{7}{23} \cdot \frac{13}{11} = \frac{455}{2024}$

d) P(at least one is vanilla) = 1 − P(all three are not vanilla) $= 1 - \frac{20}{24} \cdot \frac{19}{23} \cdot \frac{18}{22} = 1 - \frac{5}{\cancel{6}\,2} \cdot \frac{19}{23} \cdot \frac{\cancel{9}\,3}{11}$
$= 1 - \frac{285}{506} = \frac{221}{506}$

like c)

3. (10pts) The table shows the styles and gender of recording artists managed by an agent. What is the probability that a random artist from this agent:
a) is an R&B singing woman?
b) is a rock artist?
c) is a woman, given they are a country artist?
d) is a pop artist, given they are a man?
e) is not a country artist, given they are a woman?

Style	Man	Woman	Total
Pop	3	8	11
R&B	6	7	13
Rock	5	3	8
Country	5	6	11
Total	19	24	43

a) $\frac{7}{43}$ b) $\frac{8}{43}$ c) $\frac{6}{11}$ d) $\frac{3}{19}$ e) $\frac{18}{24}=\frac{3}{4}$

4. (11pts) A multiple choice test has 5 answers on every question, one of which is correct, one is acceptable, and three are incorrect. You are to select only one answer, and you get 5 points for a correct answer, 3 points for an acceptable answer, 0 points for not attempting a question, and 3 points are subtracted for an incorrect answer.
a) What is the expected value of a random guess?
b) If you can rule out one answer as incorrect, what is the expected value of a random guess?
c) If you can always rule out one answer as incorrect and randomly choose an answer among the remaining four, how many points can you expect to have on a 20-question test?

a)

outcomes		prob.
correct	5	$\frac{1}{5}$
acceptable	3	$\frac{1}{5}$
incorrect	-3	$\frac{3}{5}$

$E = 5\cdot\frac{1}{5} + 3\cdot\frac{1}{5} + (-3)\cdot\frac{3}{5} = \frac{5+3-9}{5} = -\frac{1}{5}$

b)

outcomes		prob.
correct	5	$\frac{1}{4}$
acceptable	3	$\frac{1}{4}$
incorrect	-3	$\frac{2}{4}$

$E = 5\cdot\frac{1}{4} + 3\cdot\frac{1}{4} + (-3)\cdot\frac{2}{4} = \frac{5+3-6}{4} = \frac{2}{4} = \frac{1}{2}$

c) would expect to get $20\cdot\frac{1}{2} = 10$ points

5. (11pts) A game of chance is set up as follows: A player pays \$2 and two dice are rolled. The player collects \$12 if the sum on the dice is 2, \$7 if the sum is 3 or 9, and nothing if the sum is any other number.
a) Find the expected value of this game.
b) If you play this game 40 times, how much do you expect to win or lose?
c) What is the fair price of this game?

a)

outcomes		prob.
sum is 2	10	$\frac{1}{36}$ ← (1,1)
sum is 3 or 9	5	$\frac{6}{36}$
all others	(−2)	$\frac{29}{36}$

↑ net win

(1,2),(2,1),(3,6),(4,5),(5,4)(6,3)

$36-(6+1) = 29$

$E = 10\cdot\frac{1}{36} + 5\cdot\frac{6}{36} + (-2)\frac{29}{36} = \frac{10+30-58}{36} = -\frac{18}{36} = -\frac{1}{2}$

On average, lose 0.50 per play

b) expect to lose $40\cdot 0.50 = 20$

c) $-0.50+2 = 1.50$ is fair price