Mathematical Concepts — Joysheet 3
MAT 117, Fall 2022 — D. Ivanšić

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Covers: 8.4–8.5 *Show all your work!*

1. (8pts) Peter would like to have \$3000 for a living room furniture set. How much should he deposit now in an account bearing 4.3%, compounded monthly, in order to have the desired amount in three years? How much of the \$3000 came from interest?

$A = P\left(1+\frac{r}{n}\right)^{nt}$

$3000 = P\left(1+\frac{0.043}{12}\right)^{12\cdot 3}$

$3000 = P \cdot 1.13\ldots$

$P = \frac{3000}{1.13\ldots} = 2637.53$

He deposits \$2637.53 now

$3000 - 2637.53 = 362.47$

\$362.47 is from interest.

2. (6pts) Bank of Nashville is offering a 2.36% interest rate on a savings account that is compounded quarterly, while Memphis Bank has an account at 2.35%, compounded daily. Which account is the better deal?

$Y = \left(1+\frac{r}{n}\right)^{n} - 1$

$Y_1 = \left(1+\frac{0.0236}{4}\right)^{4} - 1 = 0.0238069 \leftarrow$ more, so better deal, 2.381 % APY

$Y_2 = \left(1+\frac{0.0235}{365}\right)^{365} - 1 = 0.0237775$ 2.378 % APY

3. (10pts) You deposited \$1000 into an account bearing 3.5%, compounded quarterly. After three years, the interest rate increased to 4.5%, compounded quarterly, which inspired you to add another \$1500 to the account. How much is in the account five years after your first deposit?

$A = P\left(1+\frac{r}{n}\right)^{nt}$

$A = 1000\left(1+\frac{0.035}{4}\right)^{4\cdot 3} = 1000 \cdot 1.11\ldots = 1110.20$ add \$1500 to this to get new principal

$A = 2610.20\left(1+\frac{0.045}{4}\right)^{4\cdot 2} = 2610.20 \cdot 1.09\ldots = \$2854.58$

at the end of 5 years

4. (10pts) To save for a garage addition to their home in seven years (approximate cost \$40,000), a family makes monthly deposits into an account bearing 5.1%, compounded monthly.

a) How much should the family deposit every month to reach their goal?

b) How much would they earn in interest over the seven years?

a) $A = P\dfrac{(1+\frac{r}{n})^{nt}-1}{\frac{r}{n}}$

$40{,}000 = P \cdot \dfrac{(1+\frac{0.051}{12})^{12\cdot 7}-1}{\frac{0.051}{12}}$

$40{,}000 = P \cdot 100.695\ldots$

$P = \dfrac{40{,}000}{100.695\ldots} = 397.24$

Deposit \$397.24 monthly to reach goal

b) Total deposits $= 397.24 \cdot 12 \cdot 7 = \$33{,}368.16$

Interest earned: $40000 - 33{,}368.16 = \$6{,}631.84$

5. (16pts) At age 24, Serena got a good job and started contributing \$300 a month to a retirement account. After 8 years, she left this job for a better-paying one, so she was able to contribute \$500 a month to a retirement account. Suppose the account grew all the time at rate 8%, compounded monthly.

a) How much is in the account when Serena is 43? $24+8=32,\ 43-32=11$

b) How much of it was from deposits, and how much from interest?

a) Accumulation of 8 years of \$300 deposits:

$A = P\dfrac{(1+\frac{r}{n})^{nt}-1}{\frac{r}{n}} = 300 \cdot \dfrac{(1+\frac{0.08}{12})^{12\cdot 8}-1}{\frac{0.08}{12}} = 300 \cdot 133.86\ldots = 40{,}160.57$

This amount sits in account for 11 years as a one-time deposit

$A = P(1+\frac{r}{n})^{nt} = 40{,}160.57 \cdot (1+\frac{0.08}{12})^{12\cdot 11}$
$= 40{,}160.57 \cdot 2.40\ldots$
$= 96{,}540.76$

Accumulation of 11 years of \$500 deposits:

$A = 500\dfrac{(1+\frac{0.08}{12})^{12\cdot 11}-1}{\frac{0.08}{12}} = 500 \cdot 210.58\ldots = 105{,}290.20$

Total is $201{,}830.96$

b) Total deposits: $300\cdot 12\cdot 8 + 500 \cdot 12 \cdot 11 = 94{,}800$ Total interest: $201{,}830.96 - 94{,}800 = 107{,}030.96$

6. (10pts) You bought a home for \$140,000, and fifteen years later, you sold it for \$223,000. Assuming annual compounding, at what annual rate did this investment grow?

$A = P(1+\frac{r}{n})^{nt}$

$223{,}000 = 140{,}000\,(1+\frac{r}{1})^{1\cdot 15}$

$\dfrac{223{,}000}{140{,}000} = (1+r)^{15}$

$1.59\ldots = (1+r)^{15}$ $|\ \sqrt[15]{\ }$

$1.031\ldots = 1+r$ $|\ -1$

$r \approx 0.0315219$

Annual rate: 3.15219%