

Mathematical Concepts — Exam 1
MAT 117, Fall 2022 — D. Ivanišić

Name: _____

Show all your work!

$$I = Prt \quad A = P(1 + rt) \quad A = P \left(1 + \frac{r}{n}\right)^{nt} \quad A = P \frac{\left(1 + \frac{r}{n}\right)^{nt} - 1}{\frac{r}{n}} \quad P = PMT \frac{1 - \left(1 + \frac{r}{n}\right)^{-nt}}{\frac{r}{n}} \quad Y = \left(1 + \frac{r}{n}\right)^n - 1$$

1. (6pts) A shirt that cost \$45 is now priced at \$31.50. By how many percent was the price discounted?

2. (8pts) How much should be deposited now in an account bearing 5.2% interest, compounded weekly, in order to have \$4,000 in three years?

3. (8pts) Inflation, the rate at which costs of products increase, has clocked in at 8.3% recently. If a gallon of milk cost \$3.45 last year, how much does it cost this year? If you also pay sales tax of 7% (like in Tennessee), what is the total cost of a gallon of milk this year?

4. (8pts) Sam deposited \$1,600 into an account. After two and a half years, the account had \$1,780 in it. What simple annual interest rate did Sam earn?

5. (16pts) Married couple Penny and Cash, who have one child, file income taxes using the “married filing jointly” filing status. They earned \$104,000 in wages, \$2,100 in interest, and deposited \$9,000 into a retirement account; they paid \$8,400 in mortgage interest, \$3,100 in property taxes, \$4,400 in state income taxes and donated \$900 to charity.

- Find Penny and Cash’s gross income and adjusted gross income.
- Use the table below to first determine their taxable income (don’t forget the exemptions) and then find the tax on this income.

Tax rate	Married Filing Jointly
10%	up to \$18,550
15%	\$18,550 to \$75,300
25%	\$75,300 to \$151,900
28%	\$151,900 to \$231,450
33%	\$231,450 to \$413,350
35%	\$413,350 to \$466,950
39.6%	above \$466,950
Standard Deduction	\$12,600
Exemptions (per person)	\$4050

6. (14pts) Salma deposits \$2,000 every quarter into her retirement account starting at age 26.

- How much is in this account when Salma is 58, if it gets 7.74% interest, compounded quarterly?
- How much of the final amount is from deposits and how much from interest?

7. (32pts) True story: Queen Elizabeth II died and passed the throne and the Imperial State Crown to her son, Charles III. Made-up part: during handling of the crown, a jewel was lost and had to be replaced at a cost of \$217,000. The replacement was financed by a 20-year loan at interest rate 4.38%, compounded monthly.

a) What is the monthly payment on the loan?

b) What are the total payments over the course of the loan? How much of this amount is for interest?

c) How much of the first payment goes toward interest, and how much towards the principal?

d) What is the balance on the loan after 12 years?

8. (8pts) Investigate the effect of increased frequency of compounding: for a deposit of \$1,000 and annual interest rate of 3.21%, calculate the amount in the account after 1 year for the frequencies of compounding below.

a) Write the general formula for the amount.

b) Does compounding more often make a big difference?

Frequency: every	n	Amount after 1 year
year		
quarter		
month		
day		

Bonus. (10pts) Kiyana deposited \$200 per month into her retirement account for seven years, which earned 6.75%, compounded monthly. After her child was born, additional expenses kept her from contributing to the retirement account for the next 10 years, but the money in her retirement account continued to earn interest. How much is in the account 17 years after she started contributing?

Mathematical Concepts — Exam 2
MAT 117, Fall 2022 — D. Ivanišić

Name: _____
Show all your work!

$$\frac{a}{b} = \frac{P(E)}{1-P(E)} \quad P(E) = \frac{a}{a+b} \text{ where odds in favor of } E \text{ are } a : b \quad P(B | A) = \frac{n(A \text{ and } B)}{n(A)}$$
$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$
$$P(A \text{ or } B) = P(A) + P(B) \text{ (if } A \text{ and } B \text{ are mutually exclusive)}$$
$$P(A \text{ and } B) = P(A) \cdot P(B | A) \quad P(A \text{ and } B) = P(A) \cdot P(B) \text{ if } A \text{ and } B \text{ are independent}$$
$$E = P_1 \cdot A_1 + P_2 \cdot A_2 + \cdots + P_n \cdot A_n$$

1. (6pts) You and a friend decide to send messages in 4-character codes containing one of 29 emojis, numbers 0–9 and capital letters. How many messages are possible, if they start with a letter, the second character is an emoji, and the third character is a number, while the fourth character can be anything?

2. (6pts) A car manufacturer offers several ways to customize a certain model of a car: You can choose from 5 types of wheels, 4 interior colors, 8 exterior colors, and 3 accessory packages. Assuming the accessory package is optional, how many versions of this model can you order?

3. (10pts) The table shows the offerings of jeans in a store. What is the probability, in fraction form, that a random pair of jeans from this store:

- a) is ripped?
- b) is not ripped and skinny?
- c) is ripped or straight leg?
- d) is not ripped, given it is bell bottom?
- e) is straight leg, given it is ripped?

Type	Ripped	Not ripped	Total
bell bottom	2	3	
skinny	4	5	
straight leg	7	9	
Total			

4. (4pts) Suppose the odds in favor of all leaves falling from a particular tree by Nov. 1st are 3-to-7.

- a) What is the probability of all leaves falling from the tree by Nov. 1st?
- b) What is the probability of some leaves remaining on the tree after Nov. 1st?

5. (20pts) Write the probabilities and odds against and in favor of the following events (you can show any work needed below):

Event	probability	odds against	odds in favor
a) Drawing a picture card from a deck of cards			
b) Getting an even number on a roll of a die			
c) Getting exactly one head on two coin tosses			
d) Drawing a red queen or a black jack from a deck of cards			
e) Getting sum 8 or number 5 on the first die on a roll of two dice			

6. (12pts) The hot dog section of the supermarket refrigerator offers 24 products: 14 products contain beef, 7 products contain pork and 3 products contain both ingredients. If a product is randomly selected from the refrigerator, what is the probability it

- a) contains pork or beef?
- b) contains neither pork nor beef?

7. (12pts) A game of chance works like this: a player pays \$2 to pick a card at random from a 52-card deck. If the queen of hearts is picked, the player wins \$43. If a jack or king is picked, the player wins \$6. If any other card is picked, the player wins nothing.

- a) Determine the player's expected value.
- b) If the player plays this game 20 times, how much do they expect to win or lose?
- c) What is the fair price of this game?

8. (14pts) Simone has a 20-mile commute to school on a country two-lane road. On one trip to school, there is a 25% chance she is slowed down by farm equipment, and a 30% chance she is slowed down by a truck. Assume that encountering farm equipment or trucks on the road are independent events. What is the probability that:

- a) on a trip to school, she is slowed down by a truck and by farm equipment?
- b) on a trip to school, she is slowed down by farm equipment, but not by a truck?
- c) on a trip to school, she is slowed down by at least one type of vehicle?

- 9.** (16pts) Suppose a gladiator chooses at random two animals to fight from a group with 13 crocodiles and 9 bulls. What is the probability that
- The second one is a bull, if the first one is a bull?
 - The first is a crocodile, and the second is a bull?
 - The two animals are the same?

Bonus. (10pts) Three dice are rolled: red, white and green. Write the number of outcomes of this experiment and compute the probability that the sum on the red and white dice is 5 or the sum on the white and green dice is 6.

Final answers should have accuracy to 6 decimal places (or 4 decimal places for table-derived answers). Show some work how the mean and standard deviation are computed. *Giving only the answer will bring you few points.*

$\text{midrange} = \frac{\text{lowest value} + \text{highest value}}{2}$	$\text{range} = \text{highest value} - \text{lowest value}$
$\bar{x} = \frac{x_1 + x_2 + \cdots + x_n}{n} = \frac{\sum_i x_i}{n} = \frac{\sum_i x_i f_i}{n}$	$Z = \frac{X - \bar{x}}{s}$
$\text{margin of error} = \frac{1}{\sqrt{n}} \times 100\%$	
$s = \sqrt{\frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \cdots + (x_n - \bar{x})^2}{n - 1}} = \sqrt{\frac{\sum_i (x_i - \bar{x})^2}{n - 1}} = \sqrt{\frac{\sum_i f_i (x_i - \bar{x})^2}{n - 1}}$	

1. (8pts) A downtown beautification project is proposed for Murray’s downtown, paid for by taxpayer money. To gauge support for the idea, city officials decide to do a survey of the city’s population. Answer whether each of the following methods will produce a good, bad or questionable random sample of voters and **comment why**. Remember you are trying to decide whether every voter has an equal chance of being selected for the sample.

- a good Surveying passers-by on the court square.
 bad
 iffy

- b good Surveying people attending a basketball game at the CFSB center.
 bad
 iffy

- c good Surveying random people from the city’s property tax records.
 bad
 iffy

- d good Surveying patrons of Murray’s Dairy Queen.
 bad
 iffy

2. (9pts) The scores on an exam of a College Algebra class are shown below.

- a) Construct a grouped frequency distribution whose first class is 50–59.
- b) Are there more students in the second from top or the second from bottom class?
- c) Which class has the most students?

91, 109, 83, 92, 68, 75, 74, 86, 84, 69, 83,
65, 92, 69, 76, 100, 89, 74, 53, 64, 78, 67,
73, 101, 78, 72, 58, 74, 96, 59, 102, 71, 85,
64, 77, 96, 107

Class	Frequency

3. (18pts) Over the course of a week, a curious observer counts the number of people sitting at the computers in the main hall of Murray State's library at 4PM. She gets the numbers below.

a) Find the midrange.

14, 23, 32, 21, 11, 8, 11

b) Find the median.

c) Find the mean.

d) Find the range.

e) Find the standard deviation.

4. (25pts) A thrift store has a selection of cheaply priced items. The table below indicates the prices that appear in the store and how many items have them.

a) Draw a histogram for the data.

b) Find the mode price.

c) Find the median price.

d) Find the mean price.

e) Find the standard deviation.

Price	Frequency
\$1	25
\$2	13
\$4	11
\$7	8
\$9	12
\$10	7

5. (5pts) Men Henry from the US and Hans from Germany weigh 205 lb and 186 lb, respectively. The weights of men for their age group are normally distributed with mean 200 lb and standard deviation 13 lb for the US, and mean 182 lb and standard deviation 11 lb for Germany. Use z -scores to determine who is heavier relative to other men in their age group in their respective countries.

6. (5pts) A survey of 508 adults found that 44% of them have never owned a cat. Find the margin of error of this survey and explain what it means.

7. (13pts) The retirement age for NFL players is normally distributed with a mean of 33 years and a standard deviation of 2 years. Use the 68-95-99.7 rule (draw a picture) to find the percentage of NFL players who retire

a) between ages 31 and 35

b) before age 27

c) after age 37

d) between ages 33 and 39

8. (17pts) The annual salaries at a certain company are normally distributed with mean \$58 K and standard deviation \$12 K. Draw a picture showing which area you are computing as you answer:

- a) What percentage of employees is paid less than \$75 K?
- b) What percentage of employees is paid above \$64 K?
- c) What percentage of employees is paid between \$35 K and \$55 K?

Bonus. (10pts)

- a) For a standard normal distribution, find the 75-th percentile. Draw a picture.
- b) If scores on a test with many participants had a mean of 81 with standard deviation 9, which score will put you above 75% of the other test-takers? (*Hint: this problem is the inverse of what we usually do: on a), an area is given and we have to find the z-score. Once you have the z-score, the score on the test can easily be found in b.)*)

Mathematical Concepts — Exam 4
MAT 117, Fall 2022 — D. Ivanšić

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Show all your work!

1. (24pts) Marketing employees at an online clothing retailer are deciding which promotion to have for the holidays. The choices are: buy one, get one half off; free shipping; or twenty percent off. The employees have ranked the choices as in the table below.

a) Which choice wins the vote in a plurality election?

b) Which choice wins the vote in a plurality election with elimination?

c) Which choice is the pairwise comparison winner?

d) Which choice is the winner using Borda's method?

Perform the check on the sum of Borda points.

Votes:	3	5	1	4	6	1
1st	B	B	F	F	T	T
2nd	F	T	B	T	B	F
3rd	T	F	T	B	F	B

2. (14pts) An election for city mayor is held between candidates Fong, McDunn, Pascal and Spencer. The percentages of votes that rankings received are shown in the table.

a) Which choice wins the vote in a plurality election with elimination?

b) Which choice is the winner using Borda's method? Perform the check on the sum of Borda points.

Votes	18	15	9	17	11	16	14
1st	F	M	M	P	P	S	S
2nd	M	S	P	M	F	P	F
3rd	S	F	S	F	S	M	M
4th	P	P	F	S	M	F	P

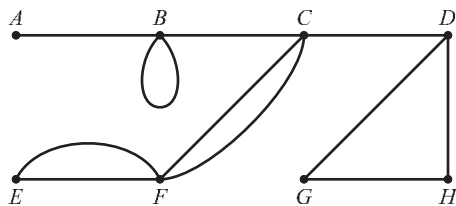
3. (12pts) A graph is shown.

a) Which vertices are adjacent to C ?

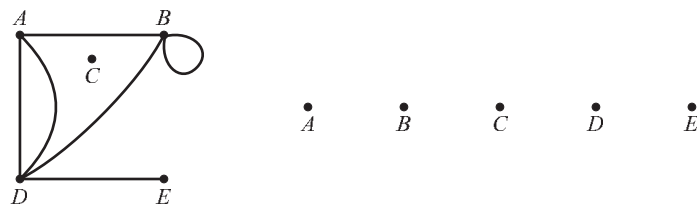
b) List the degrees of the vertices.

c) List all the bridges on the graph.

d) Give a circuit that starts and ends with F and passes through H .



4. (6pts) A graph is given and a set of vertices. Add edges to the vertices so you get a graph equivalent to the one shown.

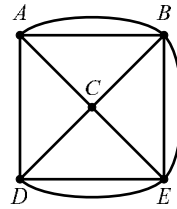
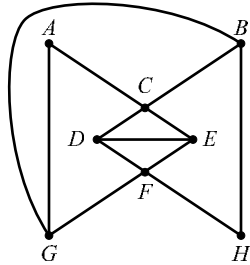
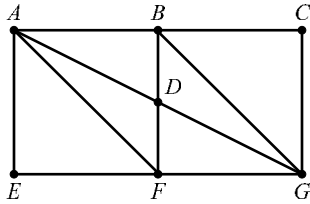


5. (16pts) For each of the following graphs:

a) State and justify whether it has an Euler path.

b) State and justify whether it has an Euler circuit.

c) If it has either an Euler path or a circuit, indicate it on the graph. Use arrows and number the edges to indicate how the Euler path or circuit goes around the graph.

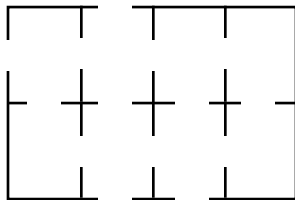


6. (14pts) Below is the floor plan of an office building, with doors joining rooms indicated.

a) Represent the floor plan as a graph (rooms are vertices, don't forget an "outside").

b) Use the graph to determine if it is possible to walk around the office building, passing through every door exactly once. If it is, draw the route.

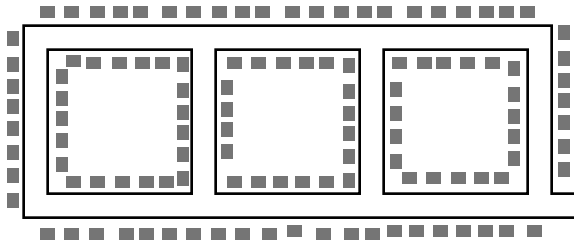
c) Is it possible to do the same as in b), and start and finish in the same room?



7. (14pts) A mail carrier has to deliver mail to the neighborhood shown in the picture. The mail carrier always walks one row of houses on one side of the street at a time.

a) Draw a graph that models the neighborhood.

b) Can the mail carrier deliver the mail to every house in the neighborhood without walking by any row of houses twice and start and end at the same place (for example, on the corner that is the exit of the neighborhood)? If so, display the route.



Bonus. (10pts) Draw a graph that has exactly two bridges and at least two even vertices.

$$I = Prt \quad A = P(1 + rt) \quad A = P \left(1 + \frac{r}{n}\right)^{nt} \quad A = P \frac{\left(1 + \frac{r}{n}\right)^{nt} - 1}{\frac{r}{n}} \quad P = PMT \frac{1 - \left(1 + \frac{r}{n}\right)^{-nt}}{\frac{r}{n}} \quad Y = \left(1 + \frac{r}{n}\right)^n - 1$$

$$\frac{a}{b} = \frac{P(E)}{1 - P(E)} \quad P(E) = \frac{a}{a+b} \text{ where odds in favor of } E \text{ are } a : b \quad P(B|A) = \frac{n(A \text{ and } B)}{n(A)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$P(A \text{ or } B) = P(A) + P(B) \text{ (if } A \text{ and } B \text{ are mutually exclusive)}$$

$$P(A \text{ and } B) = P(A) \cdot P(B|A) \quad P(A \text{ and } B) = P(A) \cdot P(B) \text{ if } A \text{ and } B \text{ are independent}$$

$$E = P_1 \cdot A_1 + P_2 \cdot A_2 + \dots + P_n \cdot A_n$$

$$\text{midrange} = \frac{\text{lowest value} + \text{highest value}}{2} \quad \text{range} = \text{highest value} - \text{lowest value}$$

$$\bar{x} = \frac{x_1 + x_2 + \dots + x_n}{n} = \frac{\sum_i x_i}{n} = \frac{\sum_i x_i f_i}{n} \quad Z = \frac{X - \bar{x}}{s} \quad \text{margin of error} = \frac{1}{\sqrt{n}} \times 100\%$$

$$s = \sqrt{\frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \dots + (x_n - \bar{x})^2}{n - 1}} = \sqrt{\frac{\sum_i (x_i - \bar{x})^2}{n - 1}} = \sqrt{\frac{\sum_i f_i (x_i - \bar{x})^2}{n - 1}}$$

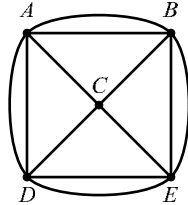
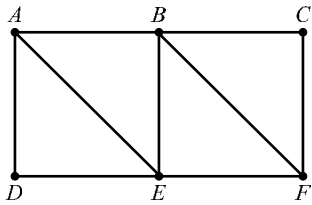
1. (24pts) A company is deciding where to take its employees for a “team-building” retreat. The choices are a cruise, Disney World and rafting. A poll of its middle managers results in the following preference rankings.

- Which choice wins the vote in a plurality election?
- Which choice wins the vote in a plurality election with elimination?
- Which choice is the pairwise comparison winner?
- Which choice is the winner using Borda’s method?

Votes:	6	2	5	2	3	6
1st	C	C	D	D	R	R
2nd	D	R	C	R	C	D
3rd	R	D	R	C	D	C

Perform the check on the sum of Borda points.

2. (12pts) For each of the following graphs:
- State and justify whether it has an Euler path.
 - State and justify whether it has an Euler circuit.
 - If it has either an Euler path or a circuit, indicate it on the graph. Use arrows and number the edges to indicate how the Euler path or circuit goes around the graph.



3. (25pts) Over one month, a city public transportation employee counts how many of its buses break down in a day and comes up with the following data.
- Draw a histogram for the data.
 - Find the mode number of broken buses.
 - Find the median number of broken buses.
 - Find the mean number of broken buses.
 - Find the standard deviation.

Buses broken	Frequency (days)
0	3
1	13
2	7
3	5
4	2

4. (12pts) Psychologists administer a certain skill test and measure the time it takes subjects to complete it. They find that the times for completion are normally distributed with mean 17 seconds and standard deviation 1.5 seconds. Draw a picture showing which area you are computing as you answer:

- a) What percentage of subjects completes the test in less than 16 seconds?
- b) What percentage of subjects completes the test in between 18 and 19 seconds?

5. (12pts) Write the probabilities and odds against and in favor of the following events (you can show any work needed below):

Event	probability	odds against	odds in favor
a) Rolling sum 5 on a roll of two dice			
b) Drawing a king or a red ace from a deck of cards			
c) Getting more heads than tails on three coin tosses			

6. (6pts) Among 57 towns surveyed, 23 had a parking garage downtown and 31 had metered parking downtown, while 46 had a parking garage or metered parking downtown. If a random town is selected, what is the probability that it has both a parking garage and metered parking downtown?

7. (12pts) The following game of chance is offered to you. The cost to play is \$1 and you roll a die. If you roll a 3, you win \$3, if you roll a 4, you win \$2, and you win nothing in all other cases.

a) Compute the expected value of this game.

b) How much would you expect to win or lose if you played 60 games?

8. (10pts) In one year, there is a 7% chance of having a devastating hurricane in Atlantic City, NJ. Assuming that the appearance of hurricanes in different years are independent events, what is the probability that Atlantic City has a devastating hurricane

a) two years in a row?

b) at least once in three consecutive years?

9. (7pts) A nice pair of pants that cost \$90 has been discounted by 25%. If purchased in Texas, where sales tax is 8.25%, what is the total cost of the pants after discount?

10. (14pts) You would like to save up for a trip to Qatar.

a) How much should you deposit every week into an account with 3% interest, compounded weekly, in order to have \$10,000 in two years?

b) How much of the final amount is from deposits and how much from interest?

11. (16pts) True story: Queen Elizabeth II died and passed the throne and the Imperial State Crown to her son, Charles III. Made-up part: a new display case with Charles' insignia had to be made at a cost of \$89,000. The replacement was financed by a 10-year loan at interest rate 5.16%, compounded monthly.

a) What is the monthly payment on the loan?

b) What are the total payments over the course of the loan? How much of this amount is for interest?

Bonus. (15pts) a) For a standard normal distribution, find the 75-th percentile. Draw a picture.

b) If scores on a test with many participants had a mean of 81 with standard deviation 9, which score will put you above 75% of the other test-takers? (*Hint: this problem is the inverse of what we usually do: on a), an area is given and we have to find the z-score. Once you have the z-score, the score on the test can easily be found in b.)*)