Mathematical Concepts — Exam 4
MAT 117, Fall 2022 — D. Ivanšić

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Show all your work!

1. (24pts) Marketing employees at an online clothing retailer are deciding which promotion to have for the holidays. The choices are: buy one, get one half off; free shipping; or twenty percent off. The employees have ranked the choices as in the table below.

a) Which choice wins the vote in a plurality election?
b) Which choice wins the vote in a plurality election with elimination?
c) Which choice is the pairwise comparison winner?
d) Which choice is the winner using Borda's method? Perform the check on the sum of Borda points.

Votes:	3	5	1	4	6	1
1st	B	B	F	F	T	T
2nd	F	T	B	T	B	F
3rd	T	F	T	B	F	B

← 20 voters

a) B 3+5=8 wins
F 1+4=5
T 6+1=7

b) F eliminated in plurality

B 8+1=9
T 7+4=11 wins

c) B 3+5+6 = 14 w
F 1+4+1 = 6

B 3+5+1 = 9
T 6+1+4 = 11 w
F 1+4+3 = 8
T 6+1+5 = 12 w

Points: B 1
F 0
T 2 overall winner

d) B: $1\cdot(4+1)+2\cdot(1+6)+3\cdot(3+5) = 43$

F: $1\cdot(5+6)+2\cdot(3+1)+3\cdot(1+4) = 34$

T: $1\cdot(3+1)+2\cdot(5+4)+3\cdot(6+1) = 43$

total 120

20 voters × 6 pts per vote = 120 ← agrees

Get a tie between B and T.

2. (14pts) An election for city mayor is held between candidates Fong, McDunn, Pascal and Spencer. The percentages of votes that rankings received are shown in the table.

a) Which choice wins the vote in a plurality election with elimination?
b) Which choice is the winner using Borda's method? Perform the check on the sum of Borda points.

Votes	18	15	9	17	11	16	14
1st	F	M	M	P	P	S	S
2nd	M	S	P	M	F	P	F
3rd	S	F	S	F	S	M	M
4th	P	P	F	S	M	F	P

← 100 voters

plurality

c) F 18 = 18 elim
M 15+9 = 24
P 17+11 = 28
S 16+14 = 30

2nd round

M 24+18 = 42
P 28 elim
S 30

3rd round

M 42+17 = 59 (M wins)
S 30+11 = 41

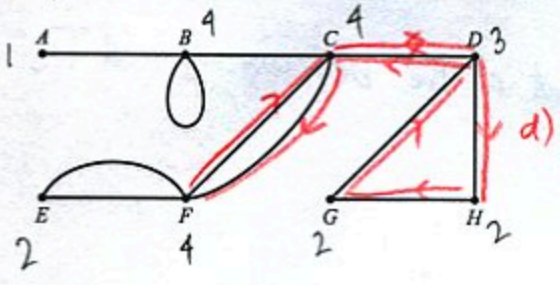
b) F $1\cdot(9+16)+2\cdot(15+17)+3(11+14)+4\cdot(18) = 236$
M $1\cdot(11)+2\cdot(16+14)+3(18+17)+4(15+9) = 272$
P $1\cdot(18+15+14)+2\cdot(0)+3(9+16)+4(17+11) = 234$
S $1\cdot(17)+2\cdot(18+9+11)+3\cdot 15+4\cdot(16+14) = 258$

M wins

1000 ↑

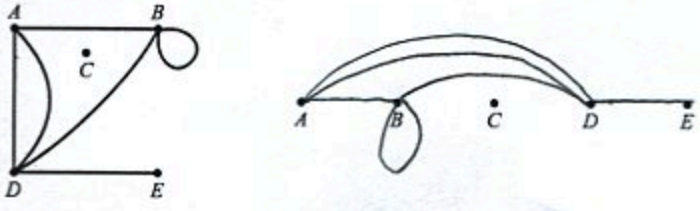
Check: voters 100 × 10 pts per voter = 1000 ← agrees

3. (12pts) A graph is shown.
a) Which vertices are adjacent to C? B, D, F
b) List the degrees of the vertices.
c) List all the bridges on the graph. BC, CD, AB
d) Give a circuit that starts and ends with F and passes through H.

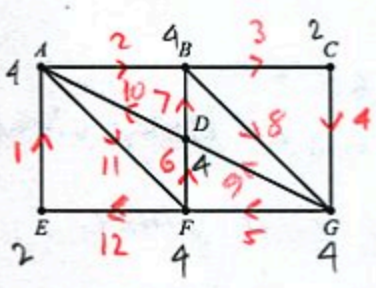


FCDHGDCF

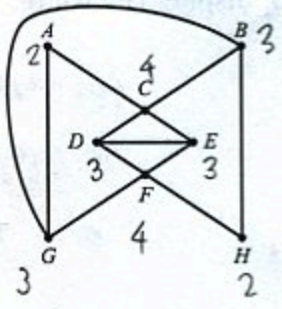
4. (6pts) A graph is given and a set of vertices. Add edges to the vertices so you get a graph equivalent to the one shown.



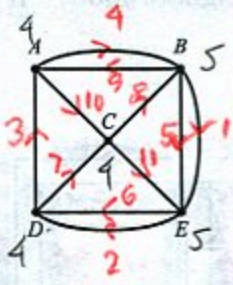
5. (16pts) For each of the following graphs:
a) State and justify whether it has an Euler path.
b) State and justify whether it has an Euler circuit.
c) If it has either an Euler path or a circuit, indicate it on the graph. Use arrows and number the edges to indicate how the Euler path or circuit goes around the graph.



All vertices even, so has Euler circuit and path



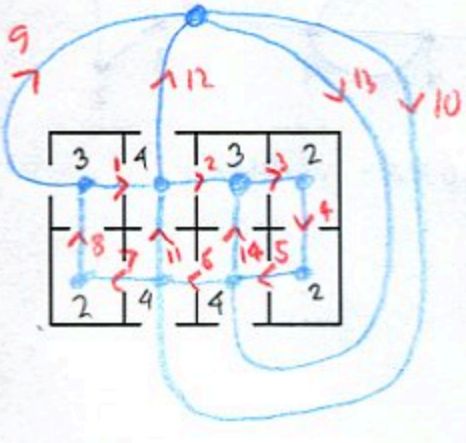
Has more than 2 odd vertices, so no Euler path and no Euler circuit



Has exactly two odd vertices so has Euler path, but no circuit.

6. (14pts) Below is the floor plan of an office building, with doors joining rooms indicated.
a) Represent the floor plan as a graph (rooms are vertices, don't forget an "outside").
b) Use the graph to determine if it is possible to walk around the office building, passing through every door exactly once. If it is, draw the route.
c) Is it possible to do the same as in b), and start and finish in the same room?

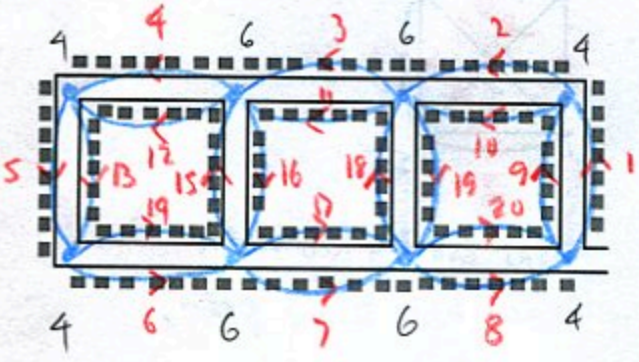
a)



b) Graph has exactly 2 odd vertices, so has an Euler path, thus, b) is possible

c) Graph has no Euler circuit, so c) is not possible

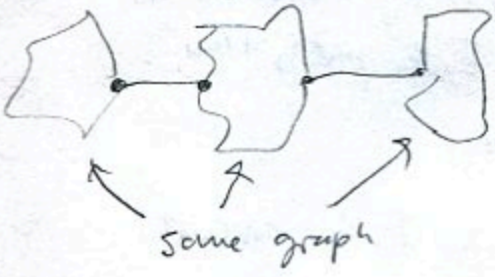
7. (14pts) A mail carrier has to deliver mail to the neighborhood shown in the picture. The mail carrier always walks one row of houses on one side of the street at a time.
a) Draw a graph that models the neighborhood.
b) Can the mail carrier deliver the mail to every house in the neighborhood without walking by any row of houses twice and start and end at the same place (for example, on the corner that is the exit of the neighborhood)? If so, display the route.

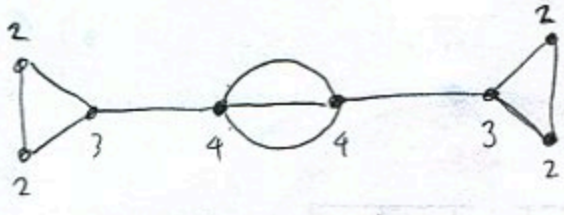


All vertices are even, so graph has an Euler circuit, and thus b) is possible.

Bonus. (10pts) Draw a graph that has exactly two bridges and at least two even vertices.

Two bridges is like





This graph has 6 even vertices.