Mathematical Concepts — Exam 2
MAT 117, Fall 2022 — D. Ivanšić

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Show all your work!

$\frac{a}{b} = \frac{P(E)}{1-P(E)}$ $\quad P(E) = \frac{a}{a+b}$ where odds in favor of E are $a : b$ $\quad P(B \mid A) = \frac{n(A \text{ and } B)}{n(A)}$

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$P(A \text{ or } B) = P(A) + P(B)$ (if A and B are mutually exclusive)

$P(A \text{ and } B) = P(A) \cdot P(B \mid A)$ $\quad P(A \text{ and } B) = P(A) \cdot P(B)$ if A and B are independent

$E = P_1 \cdot A_1 + P_2 \cdot A_2 + \cdots + P_n \cdot A_n$

1. (6pts) You and a friend decide to send messages in 4-character codes containing one of 29 emojis, numbers 0–9 and capital letters. How many messages are possible, if they start with a letter, the second character is an emoji, and the third character is a number, while the fourth character can be anything?

↑ ↑ ↑ ↑

26 29 10 65 = 26 + 29 + 10

$26 \cdot 29 \cdot 10 \cdot 65 = 490,100$

2. (6pts) A car manufacturer offers several ways to customize a certain model of a car: You can choose from 5 types of wheels, 4 interior colors, 8 exterior colors, and 3 accessory packages. Assuming the accessory package is optional, how many versions of this model can you order?

↑ ↑ ↑ ↑

5 4 8 4 (3+1 For nothing chosen)

$5 \cdot 4 \cdot 8 \cdot 4 = 640$

3. (10pts) The table shows the offerings of jeans in a store. What is the probability, in fraction form, that a random pair of jeans from this store:

a) is ripped?
b) is not ripped and skinny?
c) is ripped or straight leg?
d) is not ripped, given it is bell bottom?
e) is straight leg, given it is ripped?

Type	Ripped	Not ripped	Total
bell bottom	2	3	5
skinny	4	5	9
straight leg	7	9	16
Total	13	17	30

a) $\frac{13}{30}$ b) $\frac{5}{30} = \frac{1}{6}$ c) $\frac{2+4+7+9}{30} = \frac{22}{30} = \frac{11}{15}$ d) $\frac{3}{5}$ e) $\frac{7}{13}$

4. (4pts) Suppose the odds in favor of all leaves falling from a particular tree by Nov. 1st are 3-to-7. 3 ways it happens, 7 ways it doesn't, 10 total

a) What is the probability of all leaves falling from the tree by Nov. 1st? $\frac{3}{10}$

b) What is the probability of some leaves remaining on the tree after Nov. 1st? $\frac{7}{10}$

5. (20pts) Write the probabilities and odds against and in favor of the following events (you can show any work needed below):

	Event	probability	odds against	odds in favor
a)	Drawing a picture card from a deck of cards	$\frac{12}{52} = \frac{3}{13}$	$40:12 = 10:3$	$12:40 = 3:10$
b)	Getting an even number on a roll of a die	$\frac{3}{6} = \frac{1}{2}$	$3:3 = 1:1$	$3:3 = 1:1$
c)	Getting exactly one head on two coin tosses	$\frac{2}{4} = \frac{1}{2}$	$2:2 = 1:1$	$2:2 = 1:1$
d)	Drawing a red queen or a black jack from a deck of cards	$\frac{4}{52} = \frac{1}{13}$	$48:4 = 12:1$	$4:48 = 1:12$
e)	Getting sum 8 or number 5 on the first die on a roll of two dice	$\frac{10}{36} = \frac{5}{18}$	$26:10 = 13:5$	$10:26 = 5:13$

c) HH, HT, TH, TT

d) $\frac{2}{52} + \frac{2}{52} = \frac{4}{52}$

e) $P(\text{sum 8 OR 5 on 1st})$

$= P(\text{sum 8}) + P(\text{5 on 1st}) - P(\text{sum 8 and 5 on 1st})$

$= \frac{5}{36} + \frac{6}{36} - \frac{1}{36} = \frac{10}{36} = \frac{5}{18}$

5: 2,6 3,5 4,4 5,3 6,2 — 5,1 5,2 5,3 5,4 5,5 5,6 :6

6. (12pts) The hot dog section of the supermarket refrigerator offers 24 products: 14 products contain beef, 7 products contain pork and 3 products contain both ingredients. If a product is randomly selected from the refrigerator, what is the probability it

a) contains pork or beef?

b) contains neither pork nor beef?

a) $P(\text{pork OR beef}) = P(\text{pork}) + P(\text{beef}) - P(\text{pork AND beef})$

$= \frac{7}{24} + \frac{14}{24} - \frac{3}{24} = \frac{18}{24} = \frac{3}{4}$

b) $P(\text{neither pork nor beef}) = 1 - P(\text{pork or beef}) = 1 - \frac{3}{4} = \frac{1}{4}$

7. (12pts) A game of chance works like this: a player pays \$2 to pick a card at random from a 52-card deck. If the queen of hearts is picked, the player wins \$43. If a jack or king is picked, the player wins \$6. If any other card is picked, the player wins nothing.
a) Determine the player's expected value.
b) If the player plays this game 20 times, how much do they expect to win or lose?
c) What is the fair price of this game?

a)

	Outcomes (net gain)	prob.
queen♡	41	$\frac{1}{52}$
Jack	4	$\frac{8}{52}$
other card	-2	$\frac{43}{52}$

$E = 41 \cdot \frac{1}{52} + 4 \cdot \frac{8}{52} + (-2)\frac{43}{52} = \frac{41+32-86}{52} = -\frac{13}{52} = -\frac{1}{4}$

b) $20 \cdot (-\frac{1}{4}) = -5$
expect to lose \$5

c) $-\frac{1}{4} + 2 = 1.75$

8. (14pts) Simone has a 20-mile commute to school on a country two-lane road. On one trip to school, there is a 25% chance she is slowed down by farm equipment, and a 30% chance she is slowed down by a truck. Assume that encountering farm equipment or trucks on the road are independent events. What is the probability that:
a) on a trip to school, she is slowed down by a truck and by farm equipment?
b) on a trip to school, she is slowed down by farm equipment, but not by a truck?
c) on a trip to school, she is slowed down by at least one type of vehicle?

a) $P(\text{truck AND farm}) = P(\text{truck}) \cdot P(\text{farm}) = 0.3 \cdot 0.25 = 0.075$

b) $P(\text{farm AND not truck}) = P(\text{farm}) \cdot P(\text{not truck}) = 0.25 \cdot 0.7 = 0.175$

c) $P(\text{slowed by at least one}) = 1 - P(\text{slowed by none})$
$= 1 - P(\text{not truck AND not farm})$
$= 1 - P(\text{not truck}) \cdot P(\text{not farm})$
$= 1 - 0.7 \cdot 0.75 = 1 - 0.525 = 0.475$

9. (16pts) Suppose a gladiator chooses at random two animals to fight from a group with 13 crocodiles and 9 bulls. What is the probability that
a) The second one is a bull, if the first one is a bull?
b) The first is a crocodile, and the second is a bull?
c) The two animals are the same?

22 animals

a) $P(\text{2nd bull} \mid \text{1st bull}) = \frac{8}{21}$

b) $P(\text{1st croc AND 2nd bull}) = P(\text{1st croc})\,P(\text{2nd bull} \mid \text{1st croc})$

$= \frac{13}{22} \cdot \frac{9}{21} = \frac{39}{154}$

c) $P(\text{two animals same}) = P((\text{1st bull AND 2nd bull}) \text{ OR } (\text{1st croc AND 2nd croc}))$

$= P(\text{1st bull AND 2nd bull}) + P(\text{1st croc AND 2nd croc})$

$= P(\text{1st bull})\,P(\text{2nd bull} \mid \text{1st bull}) + P(\text{1st croc})\,P(\text{2nd croc} \mid \text{1st croc})$

$= \frac{9}{22} \cdot \frac{8}{21} + \frac{13}{22} \cdot \frac{12}{21} = \frac{12}{77} + \frac{26}{77} = \frac{38}{77}$

Bonus. (10pts) Three dice are rolled: red, white and green. Write the number of outcomes of this experiment and compute the probability that the sum on the red and white dice is 5 or the sum on the white and green dice is 6.

$P(R+W=5 \text{ OR } W+G=6) = P(R+W=5) + P(W+G=6) - P(R+W=5 \text{ AND } W+G=6)$

R W G

6 6 6

$6 \cdot 6 \cdot 6 = 216$ outcomes

R+W: 1,4; 2,3; 3,2; 4,1 — G: 6

R: 6 — W+G: 1,5; 2,4; 3,3; 4,2; 5,1

R,W,G: 1,4,2; 2,3,3; 3,2,4; 4,1,5 } 4

$= \frac{4 \cdot 6}{216} + \frac{6 \cdot 5}{216} - \frac{4}{216}$

$= \frac{24+30-4}{216} = \frac{50}{216} = \frac{25}{108}$