

$$I = Prt \quad A = P(1 + rt) \quad A = P\left(1 + \frac{r}{n}\right)^{nt} \quad A = P\frac{\left(1 + \frac{r}{n}\right)^{nt} - 1}{\frac{r}{n}} \quad P = PMT\frac{1 - \left(1 + \frac{r}{n}\right)^{-nt}}{\frac{r}{n}} \quad Y = \left(1 + \frac{r}{n}\right)^n - 1$$

1. (6pts) A shirt that cost \$45 is now priced at \$31.50. By how many percent was the price discounted?

$$45 - 31.50 = 13.50$$

$$\frac{13.50}{45} = 0.3, \quad \text{so } 30\%$$

2. (8pts) How much should be deposited now in an account bearing 5.2% interest, compounded weekly, in order to have \$4,000 in three years?

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$4000 = P\left(1 + \frac{0.052}{52}\right)^{52 \cdot 3} \quad P = \frac{4000}{1.168} = 3422.50$$

$$4000 = P \cdot 1.168$$

3. (8pts) Inflation, the rate at which costs of products increase, has clocked in at 8.3% recently. If a gallon of milk cost \$3.45 last year, how much does it cost this year? If you also pay sales tax of 7% (like in Tennessee), what is the total cost of a gallon of milk this year?

$$0.083 \cdot 3.45 = 0.29$$

$$3.74 \cdot 0.07 = 0.26$$

$$3.45 + 0.29 = 3.74$$

$$3.74 + 0.26 = 4.00$$

↑
price this year

4. (8pts) Sam deposited \$1,600 into an account. After two and a half years, the account had \$1,780 in it. What simple annual interest rate did Sam earn?

$$A = P(1 + rt)$$

$$1780 = 1600(1 + r \cdot 2.5) \quad | \div 1600$$

$$1.1125 = 1 + r \cdot 2.5 \quad | -1$$

$$0.1125 = r \cdot 2.5$$

$$r = \frac{0.1125}{2.5} = 0.045$$

4.5% simple annual interest

5. (16pts) Married couple Penny and Cash, who have one child, file income taxes using the "married filing jointly" filing status. They earned \$104,000 in wages, \$2,100 in interest, and deposited \$9,000 into a retirement account; they paid \$8,400 in mortgage interest, \$3,100 in property taxes, \$4,400 in state income taxes and donated \$900 to charity.

a) Find Penny and Cash's gross income and adjusted gross income.

b) Use the table below to first determine their taxable income (don't forget the exemptions) and then find the tax on this income.

| Tax rate | Married Filing Jointly |
|-------------------------|------------------------|
| 10% | up to \$18,550 |
| 15% | \$18,550 to \$75,300 |
| 25% | \$75,300 to \$151,900 |
| 28% | \$151,900 to \$231,450 |
| 33% | \$231,450 to \$413,350 |
| 35% | \$413,350 to \$466,950 |
| 39.6% | above \$466,950 |
| Standard Deduction | \$12,600 |
| Exemptions (per person) | \$4050 |

a) Gross income: $104,000 + 2,100 = 106,100$

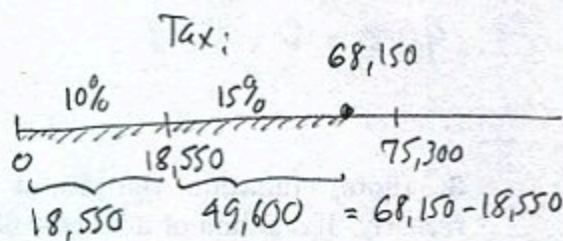
Adj. Gross income: $106,100 - 9,000 = 97,100$

b) deductions:

$$\begin{array}{r} 8,400 \\ 3,100 \\ 4,400 \\ 900 \end{array}$$

Taxable income = $97,100 - (16,800 + 3,4050) = 68,150$

$16,800$
greater than stand. ded.



Tax = $0.1 \cdot 18,550 + 0.15 \cdot 49,600 = 9,295$

6. (14pts) Salma deposits \$2,000 every quarter into her retirement account starting at age 26.

a) How much is in this account when Salma is 58, if it gets 7.74% interest, compounded quarterly?

b) How much of the final amount is from deposits and how much from interest?

a) $A = P \frac{(1 + \frac{r}{n})^{nt} - 1}{\frac{r}{n}}$

$A = 2000 \cdot \frac{(1 + \frac{0.0774}{4})^{4 \cdot 32} - 1}{\frac{0.0774}{4}}$

$= 2000 \cdot 549.08..$

$= 1,098,171.65$

b) $2000 \cdot 4 \cdot 32 = 256,000$ Total deposits

Interest: $1,098,171.65 - 256,000 = 842,171.65$

8. (8pts) Investigate the effect of increased frequency of compounding: for a deposit of \$1,000 and annual interest rate of 3.21%, calculate the amount in the account after 1 year for the frequencies of compounding below.

a) Write the general formula for the amount.

b) Does compounding more often make a big difference?

| Frequency: every | n | Amount after 1 year |
|------------------|-----|---------------------|
| year | 1 | 1,032.10 |
| quarter | 4 | 1,032.49 |
| month | 12 | 1,032.58 |
| day | 365 | 1,032.62 |

$$A = P \left(1 + \frac{r}{n}\right)^{nt}$$

$$= 1000 \left(1 + \frac{0.0321}{n}\right)^{n \cdot 1}$$

$$= 1000 \cdot \left(1 + \frac{0.0321}{n}\right)^n$$

It makes some difference, but it is small (at most additional 52 cents).

Bonus. (10pts) Kiyana deposited \$200 per month into her retirement account for seven years, which earned 6.75%, compounded monthly. After her child was born, additional expenses kept her from contributing to the retirement account for the next 10 years, but the money in her retirement account continued to earn interest. How much is in the account 17 years after she started contributing?

$$A = P \frac{\left(1 + \frac{r}{n}\right)^{nt} - 1}{\frac{r}{n}}$$

$$A = 200 \frac{\left(1 + \frac{0.0675}{12}\right)^{12 \cdot 7} - 1}{\frac{0.0675}{12}}$$

$$= 200 \cdot 107.00$$

$$= 21,400.07 \text{ amount after 7 years of contributions.}$$

This amount then sits in account for 10 years, earning interest.

$$A = P \left(1 + \frac{r}{n}\right)^{nt}$$

$$A = 21,400.07 \cdot \left(1 + \frac{0.0675}{12}\right)^{12 \cdot 10}$$

$$= 21,400.07 \cdot 1.96 \dots$$

$$= 41,951.02 \leftarrow$$

Amount in account after 17 years.

7. (32pts) True story: Queen Elizabeth II died and passed the throne and the Imperial State Crown to her son, Charles III. Made-up part: during handling of the crown, a jewel was lost and had to be replaced at a cost of \$217,000. The replacement was financed by a 20-year loan at interest rate 4.38%, compounded monthly.

- What is the monthly payment on the loan?
- What are the total payments over the course of the loan? How much of this amount is for interest?
- How much of the first payment goes toward interest, and how much towards the principal?
- What is the balance on the loan after 12 years?

$$a) P = PMT \cdot \frac{1 - \left(1 + \frac{r}{n}\right)^{-nt}}{\frac{r}{n}}$$

$$217,000 = PMT \cdot \frac{1 - \left(1 + \frac{0.0438}{12}\right)^{-12 \cdot 20}}{\frac{0.0438}{12}}$$

$$217,000 = PMT \cdot 159,69 \quad | \div 159,69$$

$$\frac{217,000}{159,69} = PMT$$

$$PMT = 1358.83$$

$$b) \text{ Total payments} =$$

$$1358.83 \cdot 12 \cdot 20 = 326,119.20$$

$$\text{Total interest} =$$

$$326,119.20 - 217,000 = 109,119.20$$

$$c) 217,000 \cdot 0.0438 \cdot \frac{1}{12} = 792.05 \quad \text{toward interest}$$

$$1358.83 - 792.05 = 566.78 \quad \text{toward principal}$$

d) Balance = present value of remaining payments ($t = 8 = 20 - 12$)

$$P = 1358.83 \cdot \frac{1 - \left(1 + \frac{0.0438}{12}\right)^{-12 \cdot 8}}{\frac{0.0438}{12}}$$

$$= 1358.83 \cdot 80.8$$

$$= 109,876.94$$

balance after 12 years