Calculus 3 — Final Exam
MAT 309, Spring 2018 — D. Ivanšić

Name: ______________________________

Show all your work!

1. (12pts) Find the equation of the plane that contains the point $A = (3, 1, -2)$ and the line given by parametric equations $x = 3 - 2t$, $y = 4$, $z = -1 + t$.

2. (20pts) Let $f(x, y) = \dfrac{1}{x^2 + y^2}$.

a) Sketch the contour map for the function, drawing level curves for levels $k = 0, \frac{1}{4}, 1, 4$.

b) At point $(1, -2)$, find the directional derivative of f in the direction of $\langle -2, -3 \rangle$. In what direction is the directional derivative the greatest? What is the directional derivative in that direction?

c) Let $\mathbf{F} = \nabla f$. Sketch the vector field $\mathbf{F}$.

Apply the fundamental theorem for line integrals to answer:

d) What is $\int_C \mathbf{F} \cdot d\mathbf{r}$, if C is the straight line from $(1, 1)$ to $(3, 0)$?

e) What is $\int_C \mathbf{F} \cdot d\mathbf{r}$, if C is a curve going from any point on level curve $k = 4$ to any point on level curve $k = 1$?

3. (14pts) The curve $\mathbf{r}(t) = \langle t+3, \cos t, \sin t \rangle$ is given, t any real number.
a) Sketch the curve in the coordinate system.
b) Find parametric equations of the tangent line to this curve when $t = 0$ and sketch the tangent line.

4. (14pts) Find and classify the local extremes for $f(x, y) = 2x^2 - 5y^2 - 2xy + 6x - 14y$.

5. (16pts) Let D be the region in the first quadrant bounded by the curves $y = x^2$, $y = 0$ and $x = 2$.

a) Sketch the region D.

b) Set up $\displaystyle\iint_D e^{x^3}\, dA$ as iterated integrals in both orders of integration.

c) Evaluate the double integral using the easier order.

6. (18pts) Sketch the region E in the first octant $(x, y, z \geq 0)$ that is inside the sphere $x^2 + y^2 + z^2 = 1$ and above the plane $z = 2y$. Then write the two iterated triple integrals that stand for $\iiint_E f\, dV$ which end in $dx\, dz\, dy$ and $dz\, dy\, dx$.

7. (14pts) Use either cylindrical or spherical coordinates to set up $\iiint_E \frac{x^2+y^2+z^2}{x^2+1}\,dV$, where E is the region inside the sphere $x^2+y^2+z^2=4$ and above the plane $z=\sqrt{2}$. Simplify the expression but do NOT evaluate the integral. Sketch the region E.

8. (10pts) Set up and simplify the set-up, but do NOT evaluate the integral: $\int_C \frac{x+y}{xy+1}\,ds$, where C is the part of the curve $y=x^3-x$ from $(-1,0)$ to $(1,0)$.

9. (20pts) Consider the region inside the circle $x^2 + y^2 = 4$ and above the lines $y = \sqrt{3}x$ and $y = -\frac{1}{\sqrt{3}}x$.

a) Draw the region.

b) Use Green's theorem to find the line integral $\int_C y^3\,dx + x^3\,dy$, where C is the boundary of the region D, traversed counterclockwise.

10. (12pts) Use differentials to estimate the change in the volume of a cylindrical can if its height decreases from 28 to 26 centimeters, and its radius increases from 14 to 15 centimeters.

Bonus (10pts) Do problem 6 for the iterated triple integral that ends in $dy\,dz\,dx$.