

Calculus 3 — Exam 3**MAT 309, Spring 2018 — D. Ivanšić****Name:** _____*Show all your work!*

1. (17pts) Let D be the region in the first quadrant bounded by the curves $y = x^2 + 1$, $x = 0$ and $y = 5$.

a) Sketch the region D .

b) Set up $\iint_D \frac{1}{\sqrt{y-1}} dA$ as iterated integrals in both orders of integration.

c) Evaluate the double integral using the easier order.

2. (17pts) Find $\iint_D xy dA$ if D is the triangle bounded by $y = 1 - x$, $y = x - 3$ and $y = 3$. Sketch the region of integration first.

3. (20pts) Use polar coordinates to find $\iint_D \frac{y}{\sqrt{x^2 + y^2}} dA$, if D is the region inside the circle $(x - 1)^2 + y^2 = 1$, outside the circle $x^2 + y^2 = 2$ and above the x -axis. Sketch the region of integration first.

4. (18pts) Sketch the region E in the first octant ($x, y, z \geq 0$) that is inside the sphere $x^2 + y^2 + z^2 = 1$ and above the plane $z = 2y$. Then write the two iterated triple integrals that stand for $\iiint_E f dV$ which end in $dx dz dy$ and $dz dy dx$.

5. (14pts) Use spherical coordinates to set up the triple integral for the volume of the region that is between the spheres $x^2 + y^2 + z^2 = 4$ and $x^2 + y^2 + z^2 = 25$, above the xy -plane, and between the planes $y = \sqrt{3}x$ and $y = -\sqrt{3}x$, the part where $y \geq 0$. Do not evaluate the integral. Sketch the region E .

6. (14pts) Use cylindrical coordinates to set up $\iiint_E \frac{x^2 + y^2 + z^2}{x^2 + y^2 + 1} dV$, where E is the region bounded by the paraboloids $z = x^2 + y^2 - 3$ and $z = 9 - x^2 - y^2$. Do not evaluate the integral. Sketch the region E .

Bonus (10pts) Do problem 4 for the iterated triple integral that ends in $dy dz dx$.