

**Calculus 3 — Exam 2**  
**MAT 309, Spring 2018 — D. Ivanšić**

**Name:** \_\_\_\_\_  
*Show all your work!*

1. (10pts) Let  $h(x, y) = x^2 + 4y^2$ .
- Find the domain of  $h$ .
  - Sketch the contour map for the function, drawing level curves for levels  $k = -1, 0, 1, 4$ . Note the domain on the picture.
  - Without computation, draw the directions of  $\nabla h(1, 0)$  and  $\nabla h(\sqrt{2}, \frac{1}{\sqrt{2}})$ . Note that these points are on the level curves you drew in b).

2. (16pts) Let  $f(x, y) = \frac{\sin^2 x}{\cos^2 y}$ .

- At point  $(\frac{\pi}{4}, \frac{\pi}{3})$ , find the directional derivative of  $f$  in the direction of  $\langle 1, 3 \rangle$ .
- In what direction is the directional derivative the greatest, and what is its value?

3. (12pts) Find the equation of the tangent plane to the hyperbolic paraboloid  $y = 2x^2 - 3z^2$  at the point  $(-1, -10, 2)$ . Simplify the equation to standard form.

4. (18pts) Let  $W = \frac{x}{y-x}$ ,  $x = te^{st}$ ,  $y = s^2 + t^2$ . Use the chain rule to find  $\frac{\partial W}{\partial t}$  when  $s = 0$ ,  $t = 2$ .

5. (12pts) The body mass index or BMI is calculated using the formula  $BMI = \frac{w}{h^2}$ , where  $w$  and  $h$  are weight and height of an individual in kilograms and meters, respectively. Use differentials to estimate the change in BMI if a 1-meter high child weighing 15 kg grows by 1.5cm in height and 0.5kg in weight.

6. (12pts) Use implicit differentiation to find  $\frac{\partial z}{\partial y}$  at the point  $(1, 1, 1)$ , if  $\sqrt{x} - \sqrt{y} + \sqrt{z} + \ln(xyz) = 1$ .

7. (20pts) Find and classify the local extremes for  $f(x, y) = x^2 - xy^2 + y^2$ .

**Bonus** (10pts) Among all rectangular boxes of volume 1, find the one with the shortest diagonal  $d$ . *Hint: minimize  $d^2$ .*