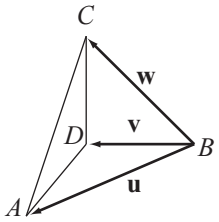
**Calculus 3 — Exam 1**
**MAT 309, Spring 2018 — D. Ivanšić**

**Name:** _____

*Show all your work!*

**1.** (11pts) Let $\mathbf{u} = \langle 3, -7, 1 \rangle$ and $\mathbf{v} = \langle 3, 0, -4 \rangle$.
a) Calculate $3\mathbf{u}$, $2\mathbf{u} - \mathbf{v}$, and $\mathbf{u} \cdot \mathbf{v}$.
b) Find a vector of length 4 in direction of $\mathbf{u}$.
c) Find the projection of $\mathbf{u}$ onto $\mathbf{v}$.

**2.** (12pts) In the picture, a tetrahedron $ABCD$ is given. All edges that meet at $D$ are perpendicular to each other and have length 4. Note that this makes $ABC$ an equilateral triangle. Draw the vectors $\mathbf{u} \times \mathbf{v}$ and $\mathbf{w} \times \mathbf{u}$ and determine their length.



**3.** (8pts) Draw the region in $\mathbf{R}^3$ described by:

$x^2 + z^2 \leq 4$, $-2 \leq y \leq 5$

**4.** (12pts) Find the equation of the plane that contains the points $A = (0, 3, -4)$, $B = (3, 1, -2)$ and $C = (4, 0, 2)$.

**5.** (16pts) This problem is about the surface $\dfrac{x^2}{4} - \dfrac{y^2}{9} - \dfrac{z^2}{1} = 1$.

a) Identify and sketch the intersections of this surface with the coordinate planes.

b) Sketch the surface in 3D, with coordinate system visible.

**6.** (14pts) The curve $\mathbf{r}(t) = \langle t, t^2 + 1, \sin t \rangle$ is given, $t$ any real number.
a) Sketch the curve in the coordinate system.
b) Find parametric equations of the tangent line to this curve when $t = 0$ and sketch the tangent line.

**7.** (13pts) Find the length of the curve $\mathbf{r}(t) = \left\langle t\cos t, t\sin t, \frac{2\sqrt{2}}{3}t^{\frac{3}{2}} \right\rangle$, $0 \leq t \leq 4$.

**8.** (14pts) Suppose the corner of a room is represented by the three coordinate planes. An egg is launched from point $(3, 7, 6)$ with initial velocity vector $\mathbf{v_0} = \langle -5, -2, 7 \rangle$.
a) Assuming gravity acts in the usual negative $z$-direction (let $g = 10$), find the vector function $\mathbf{r}(t)$ representing the egg's position.
b) On which wall or floor does the egg splatter?

**Bonus** (10pts) For every pair of skew (nonintersecting) lines in $\mathbf{R}^3$, there is a line that intersects them both and is perpendicular to both. Find this line for the two lines given parametrically below. *Hint: if $P$ is a point on one of the lines, and $Q$ on the other, what conditions have to be satisfied so that the line determined by $P$ and $Q$ is the requested line?*

$$\begin{array}{ll} x = -8 + 5t & x = 3 + 2s \\ y = -1 + t & y = -4 - 2s \\ z = -5 + 2t & z = 2 - s \end{array}$$