Write interval notation and sketch on the number line.

1. (3pts) $\{x | 4 \leq x < 7\}$

2. (3pts) $\{x | x > 9\}$

Solve the equations.

3. (3pts) $2x - 6 = 5x + 9$

4. (4pts) $1 - 2(3x - 1) - x = 4(x - 3)$

Simplify and write in standard form:

5. (4pts) $(x - 7)(x^2 + 4x - 1) - 3x^3 =$

6. (4pts) $x^2(2x + 4) - (x + 3)(x - 2) =$

Simplify and write the answer so all exponents are positive:

7. (2pts) $z^2(4z)^3 =$

8. (2pts) $\dfrac{(2x)^3}{x^7} =$

9. (3pts) $(u^2v^3)^4\, u^3v =$

10. (5pts) $(7xy^{-2})^2(x^{-4}y^2)^3 =$

11. (7pts) $\dfrac{(6w^5z^{-2})^2}{(3w^{-3}z^5)^3} =$

Factor the following.

12. (4pts) $x^2 + 7x + 12 =$ **13.** (4pts) $x^2 - 9x - 36 =$

Solve the equations.

14. (6pts) $x^2 + 1 = 2x + 16$ **15.** (6pts) $2x^2 - 10x + 4 = x^2 - 5$

1. (10pts) Let $A = (0,0)$, $B = (5,3)$, $C = (2,8)$ and $D = (-3,5)$. Draw the quadrangle $ABCD$ and then determine algebraically if it is a square by checking all sides have equal length and one angle is right.

2. (8pts) Find the equation of the circle with center $(-2,3)$ that contains the point $(1,2)$. Draw the circle.

3. (8pts) Use the graph of the function f at right to answer the following questions.

a) Find $f(3)$ and $f(0)$.

b) What is the domain of f?

c) What is the range of f?

d) What are the solutions of the equation $f(x) = \frac{1}{2}$?
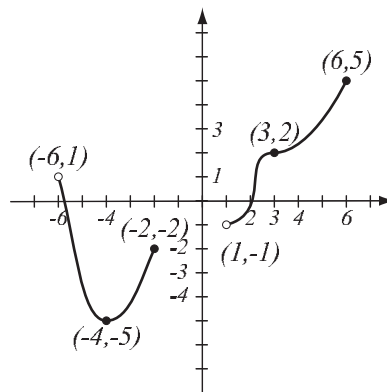
4. (12pts) The function
$f(x) = x^3 - 5x^2 + x + 2$ is given.

a) Use your calculator to accurately
its graph. Draw the graph here, and
indicate units on the axes.

b) Find all the $x-$ and $y-$intercepts
(accuracy: 6 decimal points).

c) State the domain and range.

5. (12pts) Find the domain of each function and write it using interval notation.

$$f(x) = \frac{2}{(x-4)\sqrt{x}}$$

$$g(x) = \frac{3x-2}{x^2 - 4x - 21}$$

6. (10pts) Let $h(x) = \dfrac{2x-1}{x^2 + 3x - 1}$. Find the following (simplify where appropriate).

$h\left(\dfrac{1}{2}\right) =$

$h(-2) =$

$h(\sqrt{w+1}) =$

$h(x+5) =$

1. (6pts) Find the equation of a line whose x- and y-intercepts are the same nonzero number c and which passes through point $(2, 5)$. Draw the graph of the line.

2. (10pts) Find the equation of the line (in form $y = mx + b$) passing through $(-4, 3)$ that is perpendicular to the line $3x - 4y = 12$. Draw both lines.

3. (8pts) Draw the triangle with vertices $A = (2, 1)$, $B = (5, -4)$ and $C = (-7, -5)$.
a) Find the slopes of the sides of the triangle.
b) Use slopes to check if the triangle is a right triangle.

4. (4pts) According to the Bureau of Transportation Statistics, the the number of passenger vehicles registered in the US was 193,980,000 in 2009, and 187,555,000 in 2014. What is the average rate of change of the number of passenger vehicles registered in the US from 2009 to 2014? What are the units for the average rate of change?

5. (12pts) The water bill for a family was \$59.83 in a month when it used 10 HCF (hundred cubic feet) of water. In another month, it used 15 HCF and was billed \$75.93.
a) Assuming that the water cost $C(x)$ is a linear function of the amount of water x used (in HCF), write a formula for $C(x)$.
b) What is the cost if no water is used during a month? What is the meaning of this number?
c) What is the meaning of the slope in this example?

6. (20pts) A consumer is trying to establish a relationship between the sale price P and mileage m of used 2014-2016 Camry SEs. In the table, P is the price of the car in thousands and m is the mileage in thousands. Solve the problems below with accuracy 6 decimal points.
a) Draw the scatterplot of the data. Does the relationship look linear?
b) Use two points in the scatterplot to get an equation of a line that models the relationship between P and m. Draw the line on the graph.
c) Use your calculator to find the "line of best fit" for the data. Draw the line on the graph.
d) Find the coefficient of correlation r. How strong is the linear relationship between P and m?
e) What price can the consumer expect for a car with 75,000 miles?

m	P
45	15.6
49.5	15.7
76.6	14.6
81.5	13.6
108.3	12.7
108.7	12.4

Solve the inequalities. Write your solution in interval notation.

1. (5pts) $7 < 2x + 10 \leq 9$

2. (7pts) $2 - 3x < 1$ or $7 - 2x > 10$

3. (6pts) Find the domain of the function in interval notation: $f(x) = \dfrac{\sqrt{4 - 9x}}{x^2 - 5x - 6}$.

4. (14pts) Two payroll-services companies have the following monthly cost structures. Jiffy Pay charges a $300 plus $10 per employee and Sally's Salary charges $500 per month, which includes the first 50 employees, and then $12 per employee beyond 50. Assuming a business has more than 50 employees, for which number of employees is Jiffy Pay a better deal? Solve as an inequality.

5. (14pts) A 240-mile-long road joins cities Snowden and Flakeville. At the same time, one car leaves Snowden and drives toward Flakeville, and another car, driving 6mph faster than the first car, leaves Flakeville and drives toward Snowden. After 2 hours they meet on the road.
a) What are the speeds of the cars?
b) How far from Snowden did they meet?

6. (14pts) An elementary school has 6 more girls than boys among students. Students are divided so that every teacher teaches 18 students. If the total number of students and teachers is 266, how many girls, boys and teachers does the school have?

1. (10pts) Use your calculator to accurately sketch the graph of the function
$f(x) = \dfrac{3x + 5}{x^2 + 1}$. Draw the graph here, indicate units on the axes, and solve the problems
below with accuracy 6 decimal points.
a) Find the local maxima and minima for this function.
b) State the intervals where the function is increasing and where it is decreasing.

2. (20pts) Let $f(x) = \dfrac{2}{\sqrt{x - 3}}$, $g(x) = \dfrac{x - 7}{x - 5}$. Find the following (simplify where possible):

$(f + g)(4) =$ $(fg)(-2) =$

$\dfrac{f}{g}(x) =$ $(f \circ g)(4.5) =$

$(g \circ f)(x) =$

The domain of $\dfrac{f}{g}(x)$ in interval notation

3. (8pts) Consider the function $h(x) = (x^2 - 4)^3$ and find **two** different solutions to the following problem: find functions f and g so that $h(x) = f(g(x))$, where neither f nor g are the identity function.

4. (8pts) Sketch the graph of the piecewise-defined function:

$$f(x) = \begin{cases} 3x + 5, & \text{if } -4 \le x < -1 \\ -x + 4, & \text{if } x \ge -1. \end{cases}$$

5. (14pts) A county health department is setting up COVID-19 vaccination and testing bays inside an arena. Each block of ten bays must have total area 2000 square feet. It wishes to minimize construction cost, which is same as minimizing the total length of the walls.
a) Express the total length of the walls of the building as a function of the length of one of the sides x. What is the domain of this function?
b) Graph the function in order to find the minimum. What are the dimensions of the block for which the total length of the walls is minimal? What is the minimal wall length?
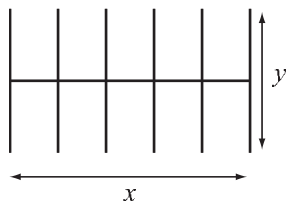
1. (21pts) For the following functions:
a) determine algebraically whether they are odd, even, or neither
b) use the calculator to draw their graphs here and verify your conclusions by stating symmetry.

$$f(x) = x^3 - 7x + 4 \qquad\qquad g(x) = x^2 + 5|x| + 3 \qquad\qquad h(x) = x - 4\sqrt[3]{x}$$

2. (16pts) Draw the graphs of $f(x) = \frac{1}{2}(x+4)^3$ and $g(x) = 3+\sqrt{-4x}$ using transformations. Explain how you transform graphs of basic functions in order to get the graphs of f and g. Indicate at least two points on each graph.

3. (10pts) Write the equation for the function whose graph has the following characteristics:
a) shape of $y = x^2$, shifted left 1 unit
b) shape of $y = \frac{1}{x}$, stretched horizontally by factor 3, then reflected about the y-axis
c) shape of $y = \sqrt[3]{x}$, stretched vertically by factor 4, then reflected about the x-axis, then shifted down 1.

4. (13pts) The graph of $f(x)$ is drawn below. On three separate graphs, sketch the graphs of the functions $f(x) - 1$, $\frac{1}{2}f(x-3)$ and $-f(-x)$ and label all the relevant points.
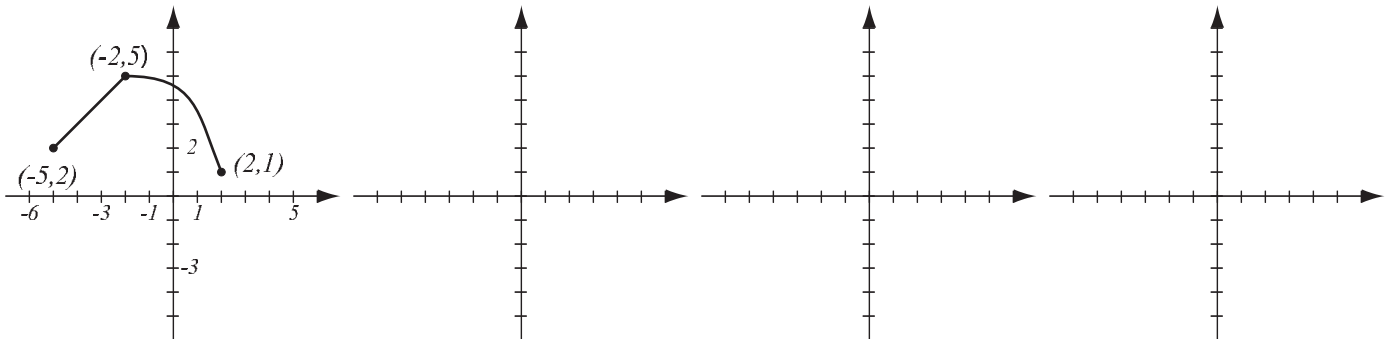
Use formulas to expand:

1. (4pts) $(u - 2v)(u + 2v) =$

2. (4pts) $(x - 5y)^2 =$

3. (5pts) $(x^2 + y^3)^2 =$

Use the *ac*-method or another method to factor. Show how you got your answer.

4. (5pts) $3x^2 - 2x - 8 =$

5. (6pts) $20x^2 + 28x - 3 =$

6. (8pts) Compute expressions with fractions by hand.

$\dfrac{2}{5} \cdot \dfrac{15}{4} =$ $\dfrac{12}{25} \div \dfrac{3}{10} =$

$\dfrac{3}{8} + \dfrac{5}{6} =$ $\dfrac{35}{18} - \dfrac{14}{45} =$

Multiply or divide the rational expressions.

7. (7pts) $\dfrac{x^2 - 4}{x^2 - 5x - 14} \cdot \dfrac{5x - 35}{2x + 4} =$

8. (7pts) $\dfrac{x^2 - 2x - 35}{2x^2 - 11x - 21} \div \dfrac{x^2 + 4x - 5}{6x + 9} =$

Add or subtract the rational expressions.

9. (6pts) $\dfrac{2x - 3}{4x^2 - 15x - 4} - \dfrac{3}{x - 4} =$

10. (8pts) $\dfrac{x - 3}{2x^2 - 7x - 30} + \dfrac{x + 1}{x^2 - 3x - 18} =$

Simplify, so that the answer is in form $a + bi$.

1. (4pts) $(3 - 4i)i + (2i)^2 =$

2. (6pts) $\dfrac{7 - 2i}{2 + i} =$

3. (4pts) Simplify and justify your answer.

$i^{146} =$

4. (8pts) The number of smartphones in storage of an electronics store is described by the function $S(x) = 0.5x^2 - 14x + 428$, where x is the number of days after November 20th.
a) On what dates did the store have 348 smartphones in storage?
b) On what date did the number of smartphones in storage bottom out?

5. (8pts) Solve the equation: $x^6 + 4x^3 - 21 = 0$

6. (6pts) Solve by completing the square.

$x^2 + 14x + 60 = 0$

7. (12pts) The quadratic function $f(x) = -x^2 + 9x - 18$ is given. Do the following without using the calculator.
a) Find the x-intercepts of its graph, if any. Find the y-intercept.
b) Find the vertex of the graph.
c) Sketch the graph of the function.

8. (12pts) Jeff's house sits on a big rectangular plot of land that is 210 by 65 meters. He wishes to enlarge it to get a rectangular plot with area 18,000 square meters by extending the 210-meter side by a certain amount and extending the 65-meter side by four times that amount. By how much should the 210- and 65-meter sides be extended to achieve the desired area?

1. (4pts) Solve the equation.

$|3x - 1| = 7$

2. (12pts) Solve the inequalities. Draw your solution and write it in interval form.

$|x - 3| > 5$ $|3x - 2| \leq 6$

Solve the equations:

**3.** (8pts) $\dfrac{x - 3}{x + 4} + \dfrac{x + 1}{x + 9} = \dfrac{x^2 + 4x - 5}{x^2 + 13x + 36}$ **4.** (8pts) $x = 3 - \sqrt{33 - x}$

5. (14pts) A marshmallow is launched from height 13 meters upwards with initial velocity 18 meters per second. Its height in meters after t seconds is given by $s(t) = -5t^2 + 18t + 13$.
a) Sketch the graph of the height function.
b) When does the marshmallow reach its greatest height, and what is that height?
c) When is the marshmallow at height 22 meters?

6. (14pts) Cora is building a shed with two rooms and a 10-ft opening between them and has enough money to build 400 feet of walls (see picture). Her goal is to maximize the enclosed area.
a) Express the area of the shed as a function of one of the sides of the rectangle. What is the domain of this function?
c) Sketch the graph of the area function in order to find the maximum (no need for the graphing calculator — you should already know what the graph looks like). What are the dimensions of the shed that has the greatest area and what is the greatest area possible?
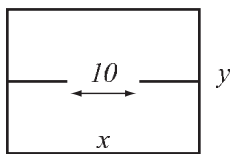
1. (6pts) The graph of a function f is given.
a) Is this function one-to-one? Justify.
b) If the function is one-to-one, find the graph
of f^{-1}, labeling the relevant points.
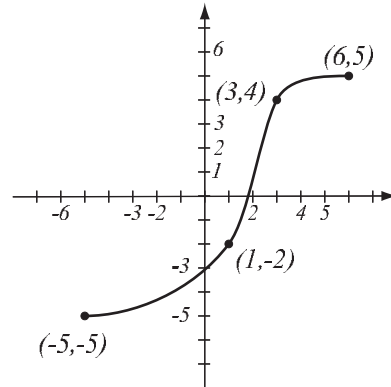
2. (12pts) Let $f(x) = 1 - \dfrac{3}{x-2}$. Find the formula for f^{-1}. Find the ranges of f and f^{-1}.

3. (8pts) Evaluate without using the calculator:

$\log_6 36 =$ $\log_7 \frac{1}{343} =$ $\log_8 32 =$ $\log_{a^2} \sqrt{a} =$

4. (4pts) Use the change-of-base formula
and your calculator to find $\log_{27} 4$ with ac-
curacy 6 decimal places. Show how you
obtained your number.

5. (12pts) Investigate the effect of increased frequency of compounding: for a deposit of $10,000 and annual interest rate of 3.75%, calculate the amount in the account after 1 year for the frequencies of compounding below.
a) Write the general formula for the amount, replacing the variables by numbers, if known.
b) Use the table feature on your calculator to quickly compute amounts after 1 year.
c) Does compounding more often make a big difference?

Frequency: every	n	Amount after 1 year
year		
quarter		
month		
day		
hour		
second		

6. (3pts) Find the domain of $f(x) = \ln(4 - 6x)$.

7. (8pts) The cost per person of a field trip for x students is given by $C(x) = \dfrac{200 + 32x}{x}$, where C is in dollars.
a) Find the cost per person if 24 or 32 students go.
b) Find a formula for the inverse function and explain what it represents.
c) How many students need to go so that cost per person is $52? $37?

8. (7pts) Using transformations, draw the graph of $f(x) = e^{-x} + 4$. Explain how you transform the graph of a basic function in order to get the graph of f. Show at least one point on the graph, and asymptotes to the graph, if any.

1. (5pts) If $\log_a 5 = u$ and $\log_a 9 = v$, express in terms of u and v:

$\log_a 405 =$ $\qquad\qquad\qquad\qquad\qquad$ $\log_a \dfrac{25}{3} =$

2. (11pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$\log_2 \left(32u^7 v^4\right) =$

$\log_{1000} \dfrac{10\sqrt[4]{x^3 y^3}}{x^{\frac{5}{8}} y^8} =$

3. (12pts) Write as a single logarithm. Simplify if possible.

$3\ln(2y^4) - \frac{1}{3}\ln(64y^{18}) + 4\ln x =$

$\log_7(x^2 - 3x - 18)^2 - 3\log_7(x - 6) + 2\log_7(x + 3) =$

Solve the equations.

4. (5pts) $6^{3x+1} = \dfrac{1}{216^{x-2}}$

5. (7pts) $5^{x-4} = 9^{4x+3}$

6. (8pts) $\log_4(x - 1) + \log_4(x + 5) = 2$

7. (12pts) The town of Snakehead had 44,000 inhabitants in 2016 and 51,000 in 2019. Assume the population of Snakehead grows exponentially.
a) Write the function describing the number $P(t)$ of people in Snakehead t years after 2016. Then find the exponential growth rate for this population.
b) Graph the function.
c) According to this model, when will the population reach 100,000?