College Algebra — Joysheet 11	**Name:** Saul Ocean
MAT 140, Spring 2021 — D. Ivanšić	*Covers: 5.4–5.6* *Show all your work!*

1. (5pts) If $\log_a 5 = u$ and $\log_a 9 = v$, express in terms of u and v:

$\log_a 405 = \log_a(5\cdot 81) = \log_a(5\cdot 9^2)$
$= \log_a 5 + \log_a 9^2$
$= \log_a 5 + 2\log_a 9$
$= u + 2v$

$\log_a \frac{25}{3} = \log_a 25 - \log_a 3$
$= \log_a 5^2 - \log_a 9^{1/2}$
$= 2\log_a 5 - \frac{1}{2}\log_a 9$
$= 2u - \frac{1}{2}v$

2. (11pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$\log_2(32u^7v^4) = \log_2 32 + \log_2 u^7 + \log_2 v^4$
$= 5 + 7\log_2 u + 4\log_2 v$

$\log_{1000} \frac{10\sqrt[4]{x^3}y^3}{x^{\frac{5}{8}}y^8} = \log_{1000} 10 + \log_{1000} x^{\frac{3}{4}} + \log_{1000} y^3 - \log_{1000} x^{\frac{5}{8}} - \log_{1000} y^8$
$= \frac{1}{3} + \frac{3}{4}\log_{1000} x + 3\log_{1000} y - \frac{5}{8}\log_{1000} x - 8\log_{1000} y$

$1000^{?} = 10$
$= \sqrt[3]{1000} = 1000^{\frac{1}{3}}$

$= \frac{1}{3} + \frac{1}{8}\log_{1000} x - 5\log_{1000} y$

3. (12pts) Write as a single logarithm. Simplify if possible.

$3\ln(2y^4) - \frac{1}{3}\ln(64y^{18}) + 4\ln x = \ln(2y^4)^3 - \ln(64y^{18})^{\frac{1}{3}} + \ln x^4$

$= \ln \frac{(2y^4)^3 \cdot x^4}{64^{\frac{1}{3}}(y^{18})^{\frac{1}{3}}} = \ln \frac{\overset{2}{\cancel{8}}y^{12}x^4}{\cancel{4}y^6} = \ln(2x^4y^{\overset{12-6}{6}})$

$\log_7(x^2 - 3x - 18)^2 - 3\log_7(x-6) + 2\log_7(x+3) =$

$= \log_7((x-6)(x+3))^2 - \log_7(x-6)^3 + \log_7(x+3)^2$

$= \log_7 \frac{(x-6)^{\cancel{2}}(x+3)^2(x+3)^2}{(x-6)^{\cancel{3}\,1}} = \log_7 \frac{(x+3)^4}{x-6}$

Solve the equations.

4. (5pts) $6^{3x+1} = \dfrac{1}{216^{x-2}}$

$$6^{3x+1} = \frac{1}{(6^3)^{x-2}} = \frac{1}{6^{3x-6}}$$

$$6^{3x+1} = 6^{-3x+6}$$

$$3x+1 = -3x+6$$

$$6x = 5, \quad x = \frac{5}{6}$$

5. (7pts) $5^{x-4} = 9^{4x+3}$ $\quad |\ln$

$$\ln 5^{x-4} = \ln 9^{4x+3}$$

$$(x-4)\ln 5 = (4x+3)\ln 9$$

$$x\cdot\ln 5 - 4\ln 5 = 4x\ln 9 + 3\ln 9$$

$$x\ln 5 - 4x\ln 9 = 4\ln 5 + 3\ln 9$$

$$x(\ln 5 - 4\ln 9) = 4\ln 5 + 3\ln 9$$

$$x = \frac{4\ln 5 + 3\ln 9}{\ln 5 - 4\ln 9} = -1.81482$$

6. (8pts) $\log_4(x-1) + \log_4(x+5) = 2$

$$\log_4((x-1)(x+5)) = 2 \quad | 4^{-}$$

$$4^{\log_4 (x-1)(x+5)} = 4^2$$

$$(x-1)(x+5) = 16$$

$$x^2 + 4x - 5 = 16$$

$$x^2 + 4x - 21 = 0$$

$$(x+7)(x-3) = 0$$

$$x = \cancel{-7},\ 3$$

gives neg. no in logarithm

7. (12pts) The town of Snakehead had 44,000 inhabitants in 2016 and 51,000 in 2019. Assume the population of Snakehead grows exponentially.
a) Write the function describing the number $P(t)$ of people in Snakehead t years after 2016. Then find the exponential growth rate for this population.
b) Graph the function.
c) According to this model, when will the population reach 100,000?

a) $P(t) = 44e^{kt}$

$$51 = P(3) = 44e^{k\cdot 3}$$

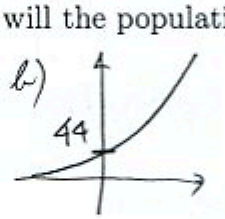
$$\frac{51}{44} = e^{k\cdot 3} \quad |\ln$$

$$\ln\frac{51}{44} = 3k$$

$$k = \frac{\ln\frac{51}{44}}{3} = 0.049212$$

$$P(t) = 44e^{0.049212t}$$

b)



c) $P(t) = 100$

$$44e^{0.049\ldots t} = 100$$

$$e^{0.049\ldots t} = \frac{100}{44} \quad \frac{25}{11}$$

$$0.049\ldots t = \ln\frac{25}{11}$$

$$t = \frac{\ln\frac{25}{11}}{0.049} = 16.682528$$

About 17 years after 2016, in 2033.